

# Round-Optimal Secure Multiparty Computation with Honest Majority

Prabhanjan Ananth    Arka Rai Choudhuri    **Aarushi Goel**    Abhishek Jain



# Secure Multiparty Computation



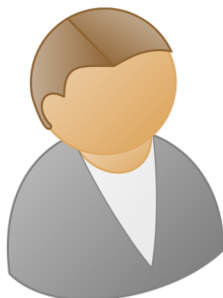
$x_1$



$x_2$

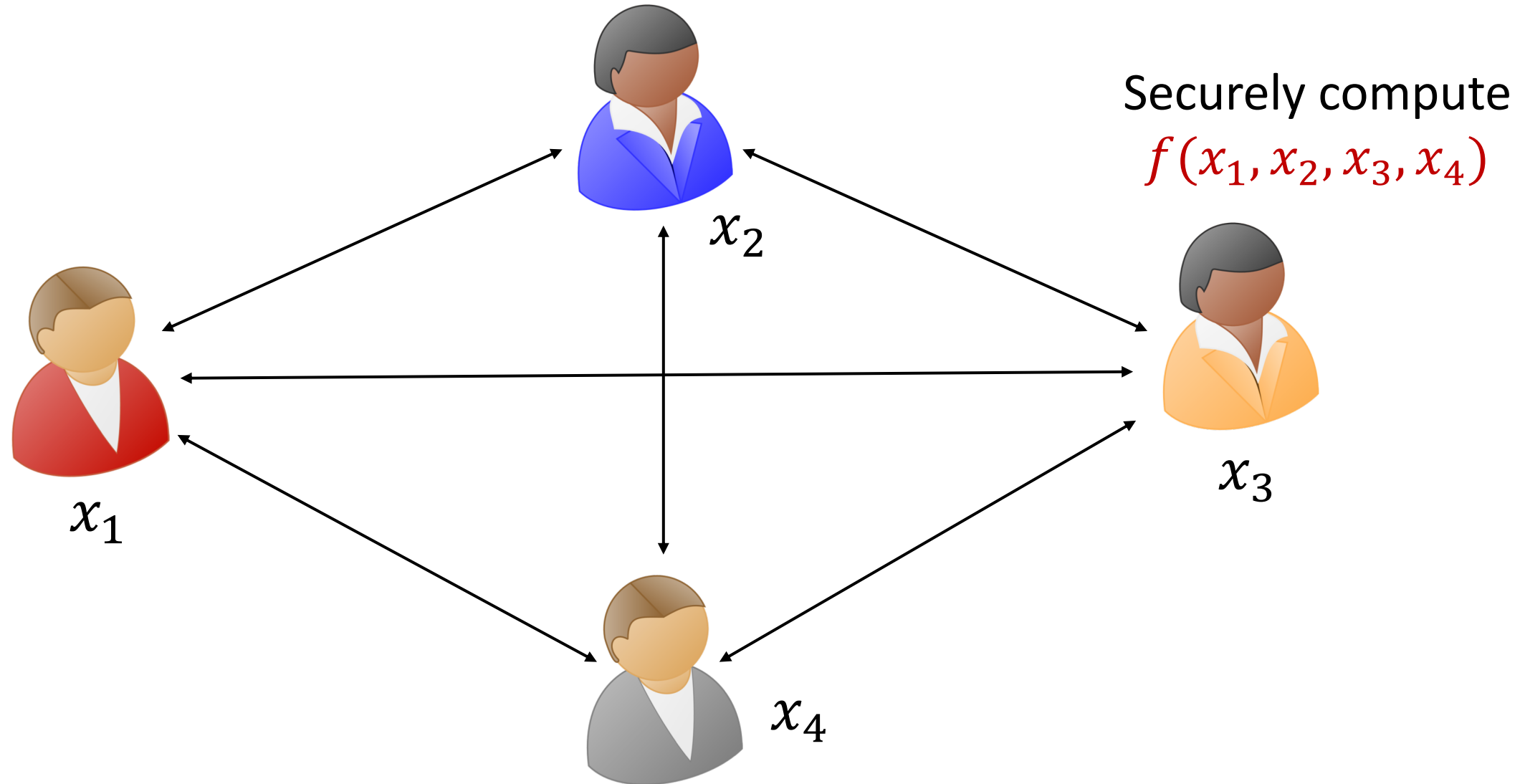


$x_3$

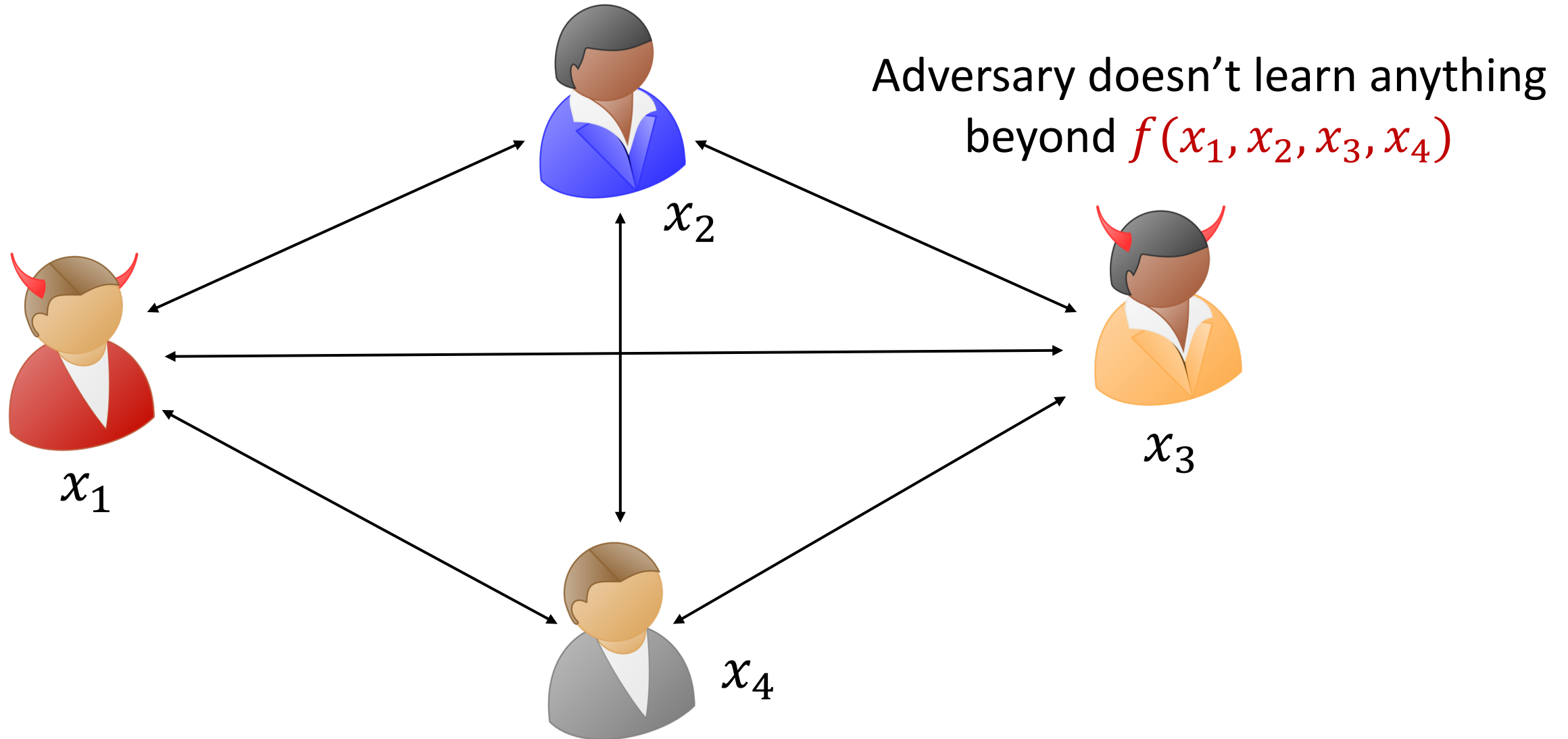


$x_4$

# Secure Multiparty Computation



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# Honest Majority MPC

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- **Fairness** and **Guaranteed output delivery** can be achieved.
- **UC security** without external trusted setups
- **Round complexity** lower bounds of dishonest majority do not apply.  
4 rounds necessary for dishonest majority in the plain model [Garg-Mukherjee-Pandey-Polychroniadou16]

# Problem Statement

What is the exact **round complexity** of honest majority MPC in the plain model?

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Guaranteed output delivery  $\Rightarrow$  Fairness

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- **Guaranteed output Delivery:**

Honest parties always learn the output even if some parties abort prematurely.

Guaranteed output delivery  $\Rightarrow$  **Fairness**

**Goal:** Develop round optimal protocols in these settings.

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- [Benhomouda-Lin17, Garg-Srinivasan17]:  $t < N$  semi-honest corruptions based on **OT**. Malicious corruptions in the CRS model.



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Impossible for dishonest majority [Garg- Mukherjee-Pandey-Polychroniadou16]

Open even in **semi-honest** case from assumptions weaker than OT.

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- [Gordon-Liu-Shi'15]: **Impossibility** of **two-round** broadcast channel protocols against **fail-stop** corruptions.

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Both questions open regardless of assumptions.

# Our Results: Security with Abort

Two round MPC for general functionalities in the plain model, assuming one-way functions.



# Our Results: Guaranteed Output Delivery

**Fail-Stop Corruptions:** Two round MPC for general functions:

**Broadcast** channel protocol in the **bare-public-key model**, assuming **PKE**.

**Point-to-point** channel protocol in the **plain model**, assuming **OT**.

# Our Results: Guaranteed Output Delivery

Fail-Stop Corruptions:

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Three round MPC from one-way functions in the plain model.

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# Our Results: Guaranteed Output Delivery

**Fail-Stop Corruptions:** Two round MPC for general functions:

Broadcast channel protocol in the bare-public-key model, assuming PKE.

Point-to-point channel protocol in the plain model, assuming OT.

**Malicious Corruptions:** Three round MPC for general functions:

Broadcast channel protocol in the plain model, assuming Zaps and PKE.

# Security with Abort against Malicious Adversaries

[Garg-Srinivasan17]

A compiler from any polynomial round MPC protocol to a two round protocol using two round UC secure OT.

[Garg-Srinivasan17]

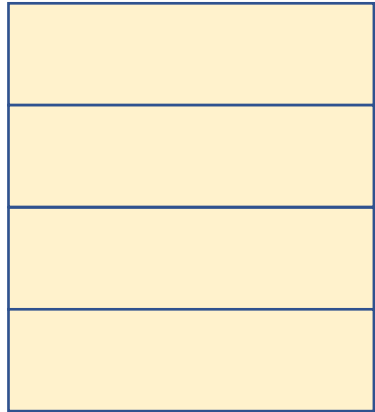
A compiler from any polynomial round MPC protocol to a two round protocol using two round UC secure OT.

**Starting Idea:** Leverage honest majority to remove OT.

[Garg-Srinivasan17]

Use of OT in [GS17]

# [Garg-Srinivasan17]



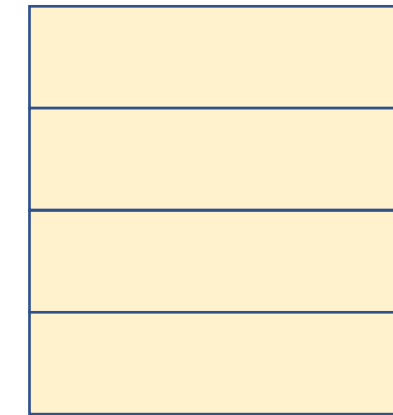
Any polynomial  
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## Use of OT in [GS17]

Start with any dishonest  
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OT over broadcast channels



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Any polynomial  
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OT+GC



Two-round MPC  
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## Use of OT in [GS17]

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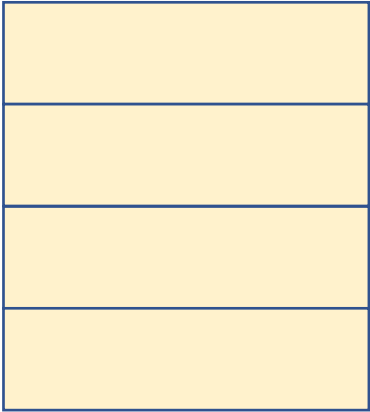
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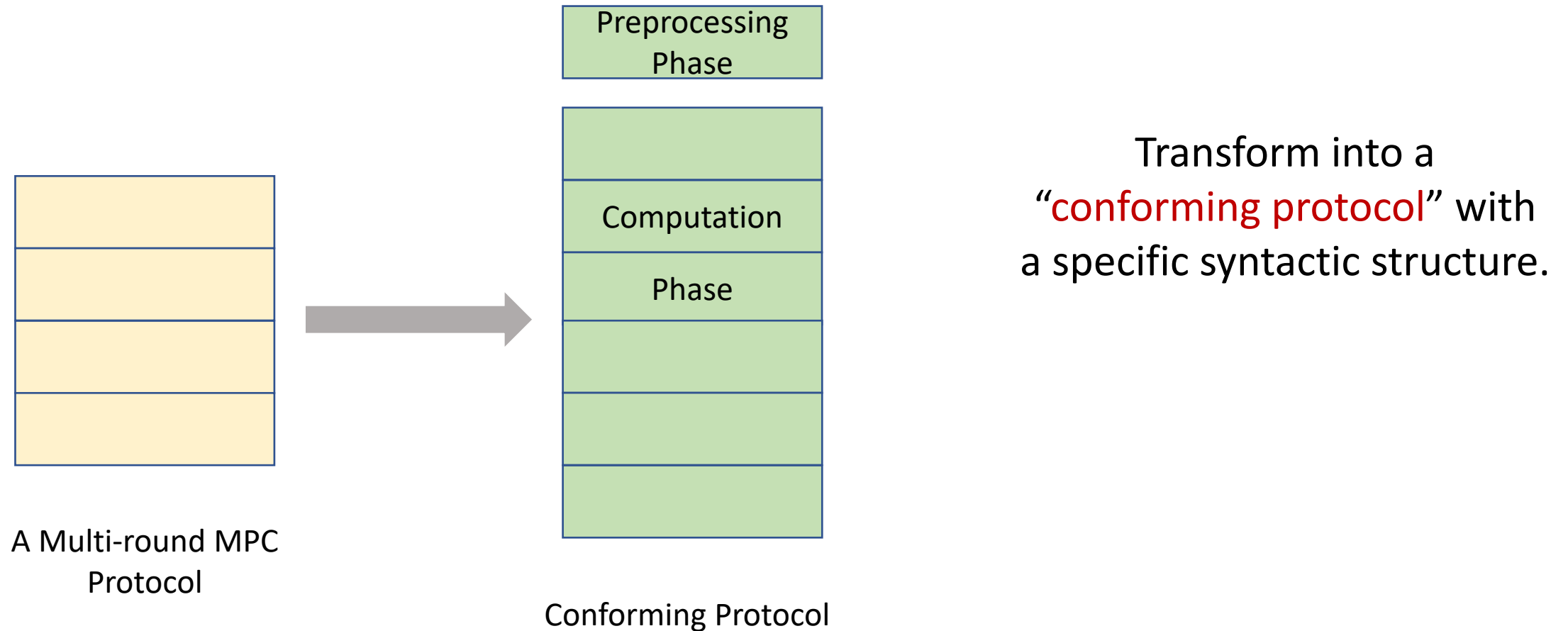
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# Recap of [Garg-Srinivasan17]



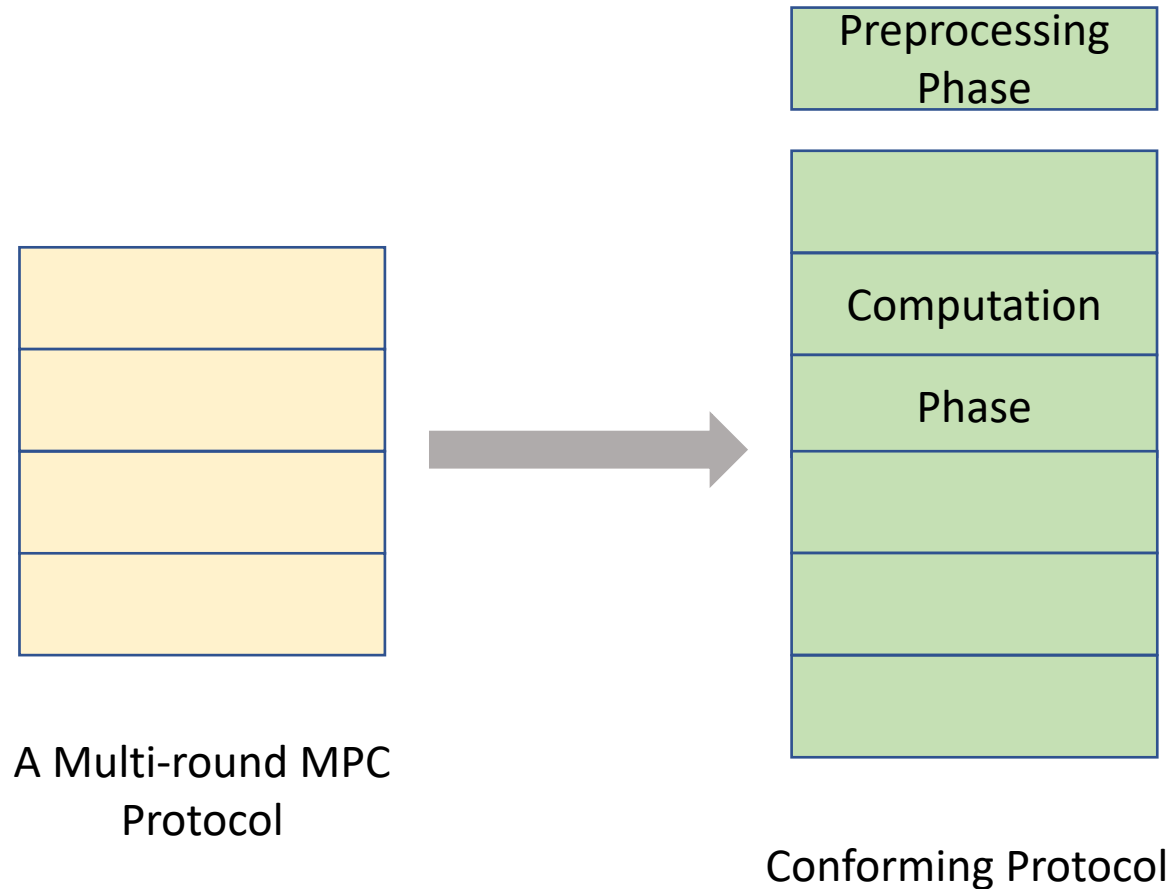
A Multi-round MPC  
Protocol

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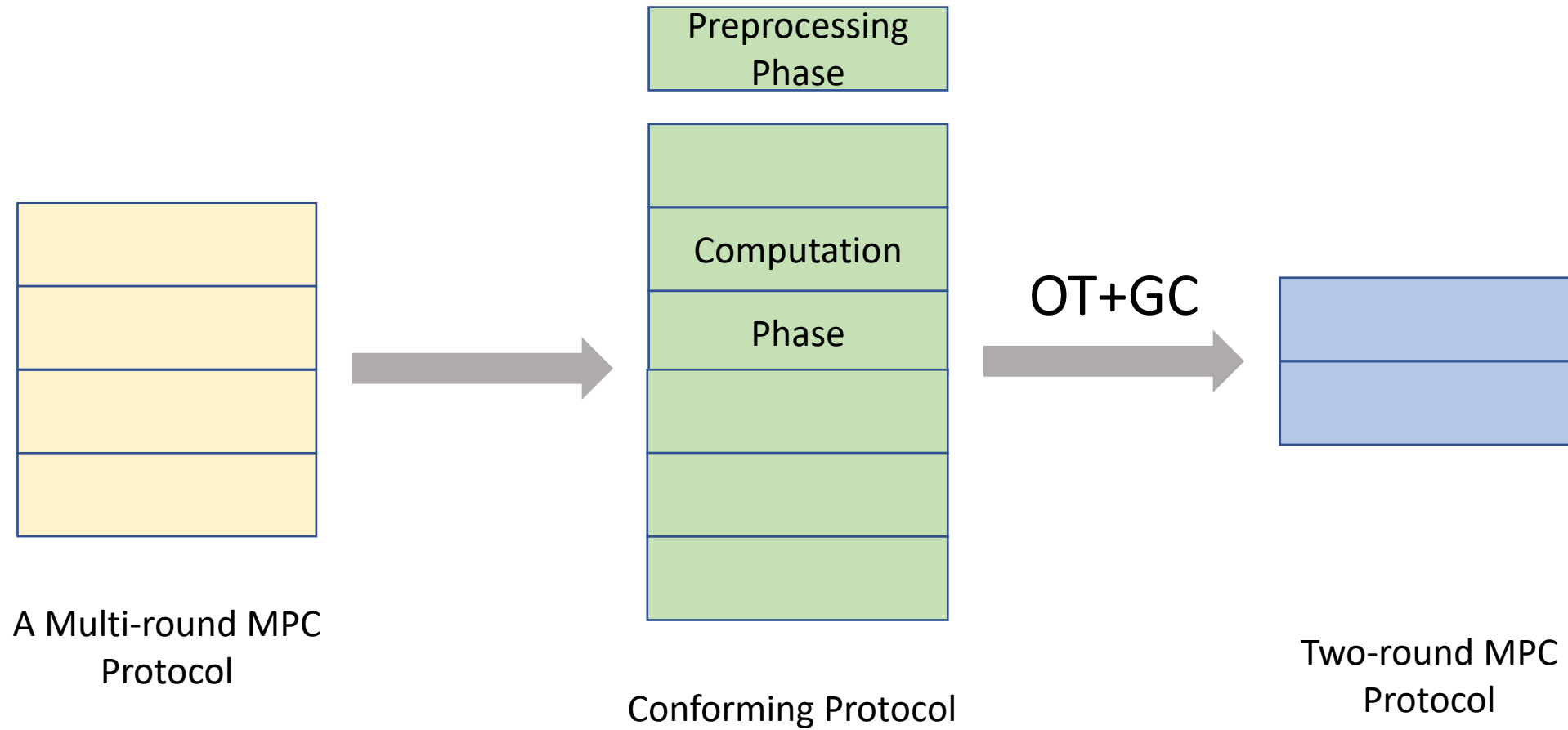


Computation Phase:

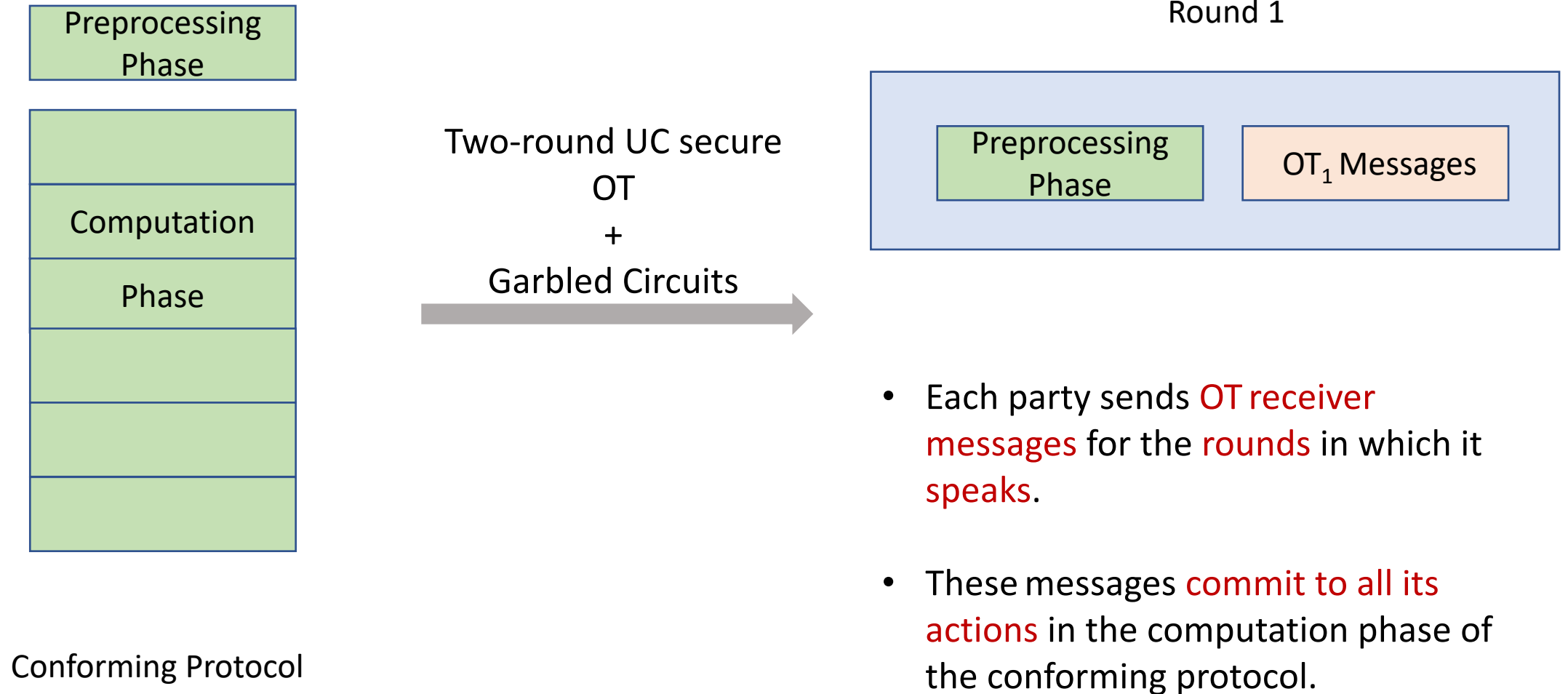
Only a **single bit** is broadcasted by a single party (**speaker**) in each round.

All other parties are **listeners** for that round.

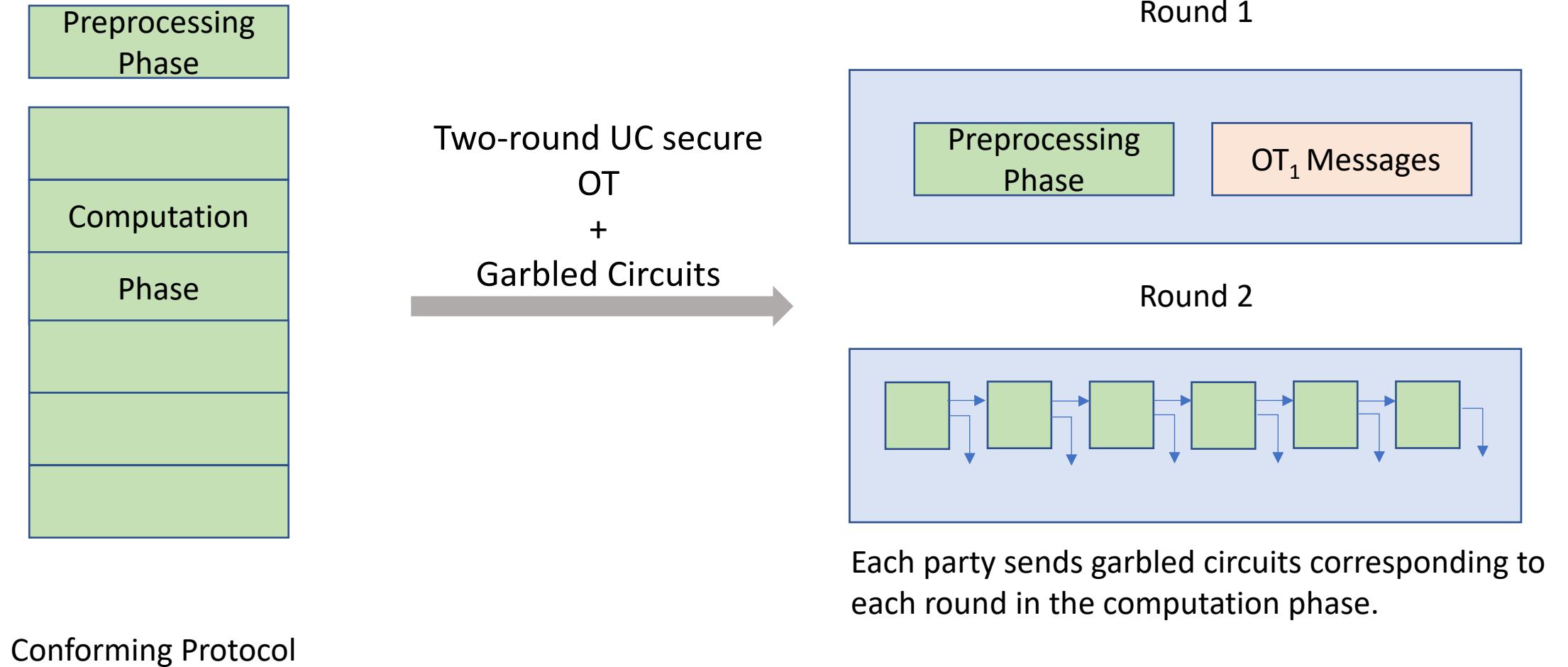
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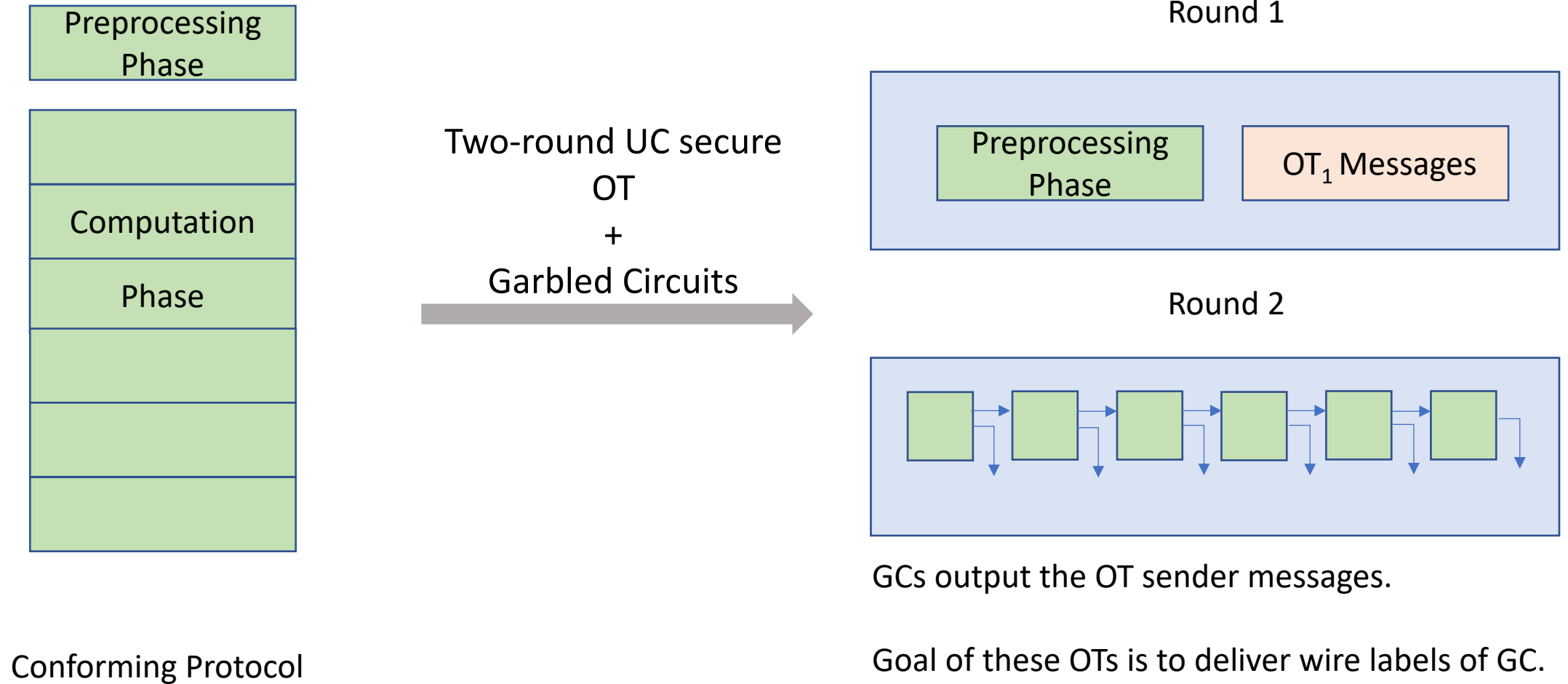
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# Our Strategy: Challenge 2

	Use of OT in [GS17]	Our approach	Challenges
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2	Compile it into a 2 round protocol using OT and Garbled circuits	Leverage honest majority to replace OT	How to achieve <b>OT functionality without OT</b> ?

# New Gadget for OT: Multi-party OT

Multi-party protocol.

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OT functionality for sender inputs  $(m_0, m_1)$  and receiver input  $(b)$  can be represented as a degree 2 polynomial in  $\mathbb{F}_2$ .

$$m_b = m_0(1 + b) + m_1(b)$$

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Multi-party protocol.

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Later: How to implement

# Our Strategy: Challenge 1

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# Compressing Private Channel Protocols

Perfectly Secure
Honest Majority
Protocol

Uses both broadcast and  
private channels.

# Compressing Private Channel Protocols

Setup Phase

Perfectly Secure

Honest Majority

Protocol

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Setup Phase

Exchange one-time pads to  
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Honest Majority

Protocol

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Perfectly Secure

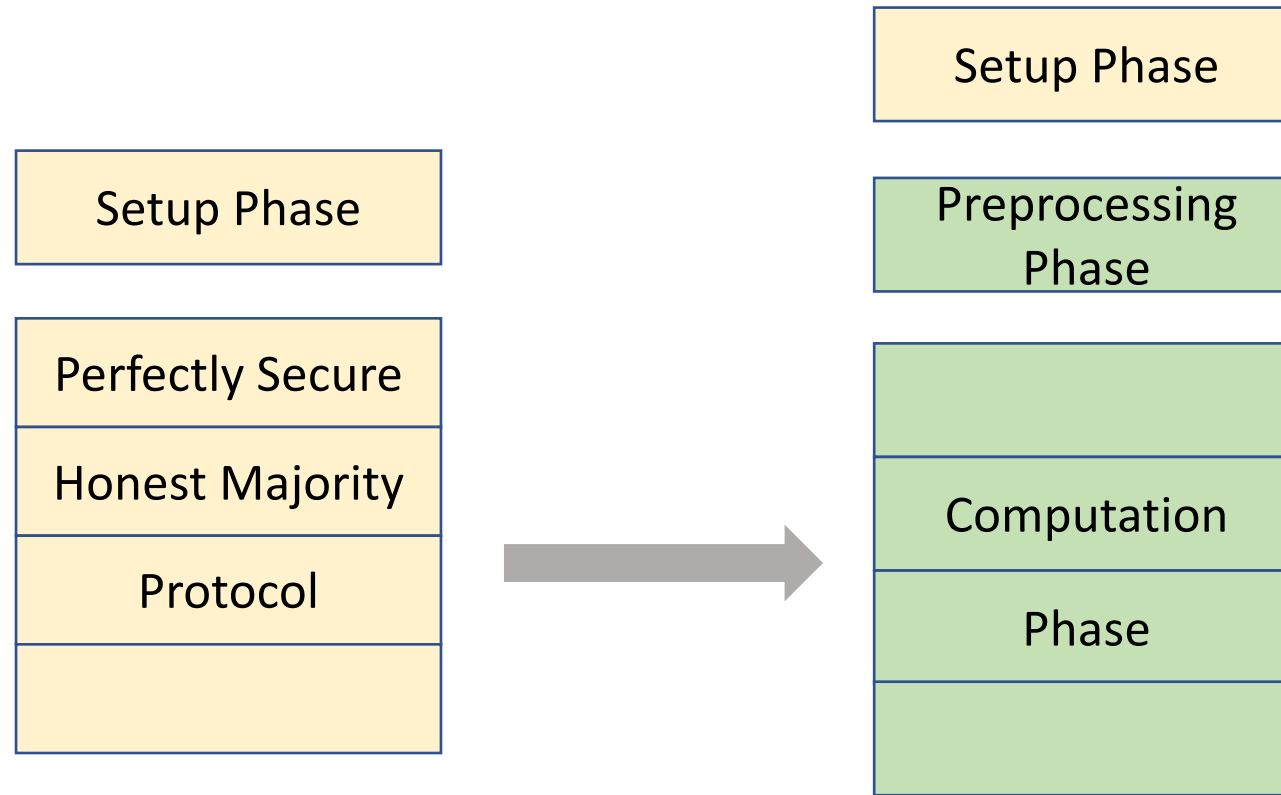
Honest Majority

Protocol

Only uses broadcast channels



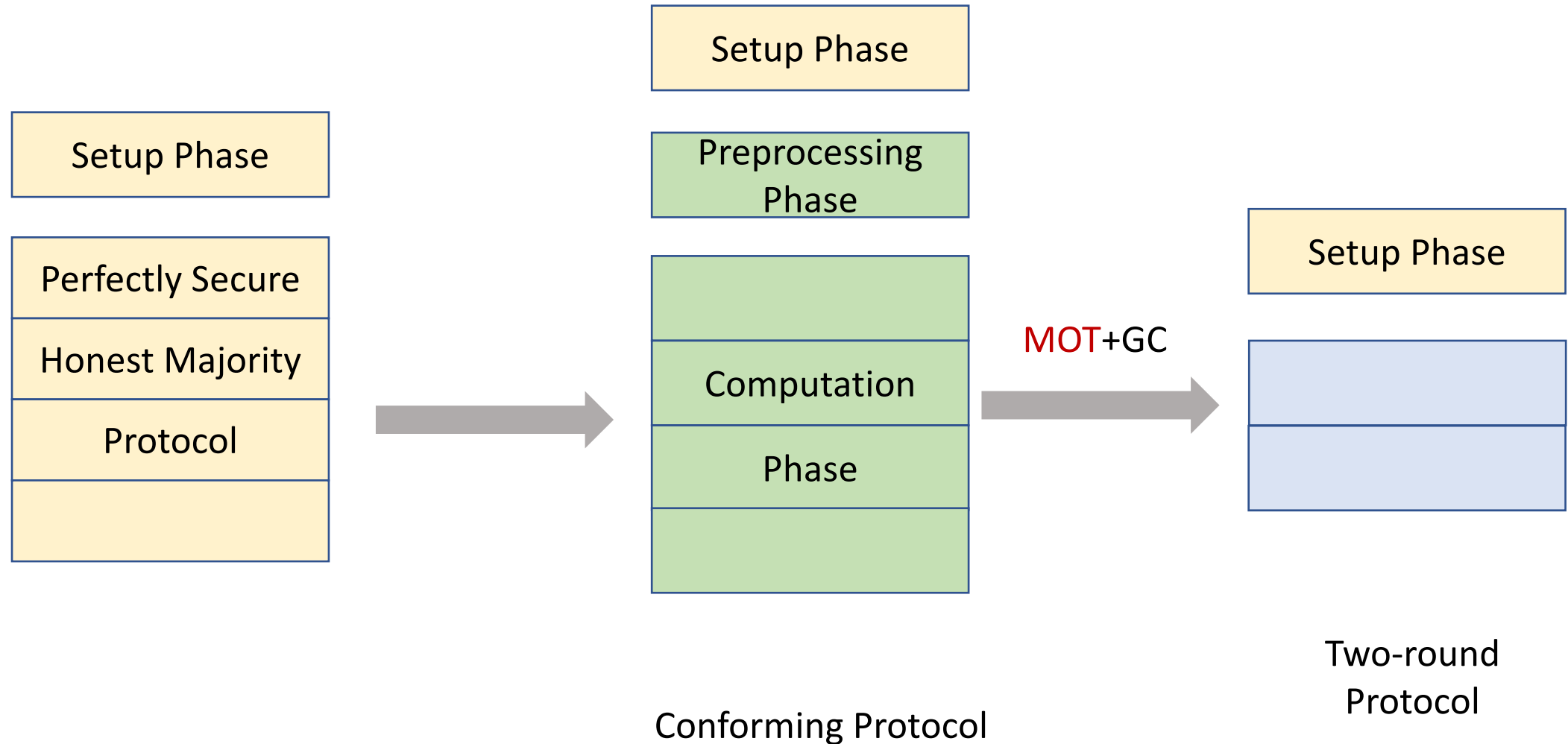
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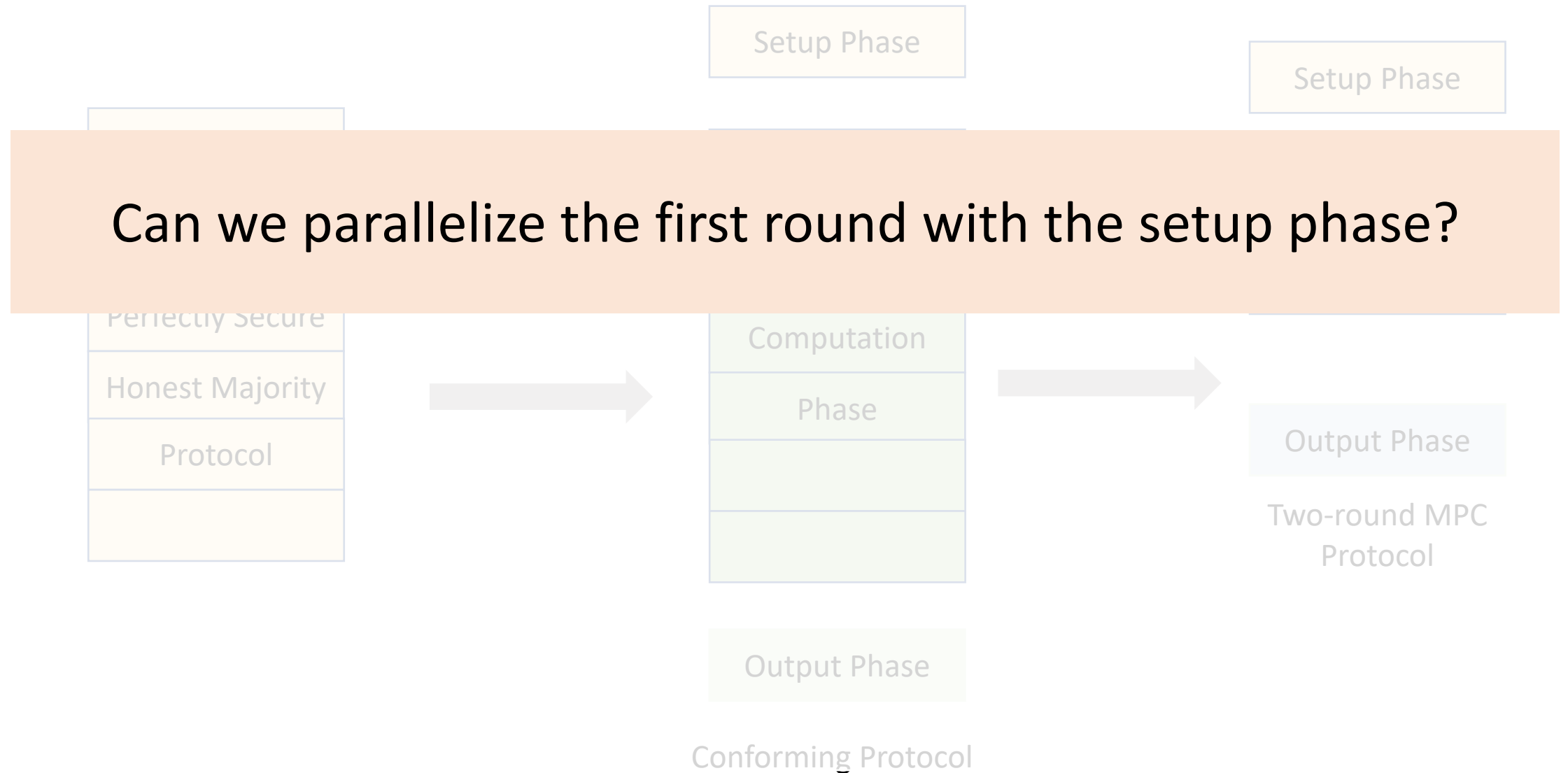
Conforming Protocol

Transform to a  
conforming protocol  
with a **setup phase**

# Compressing Private Channel Protocols

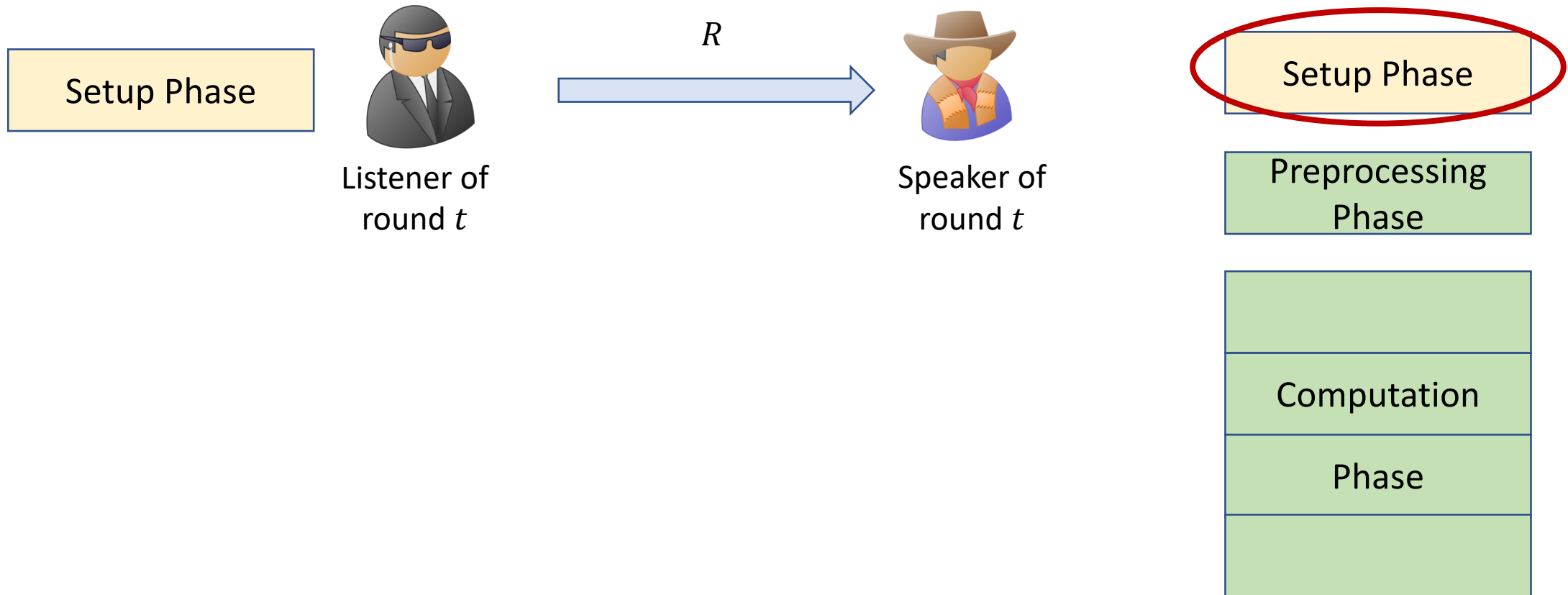


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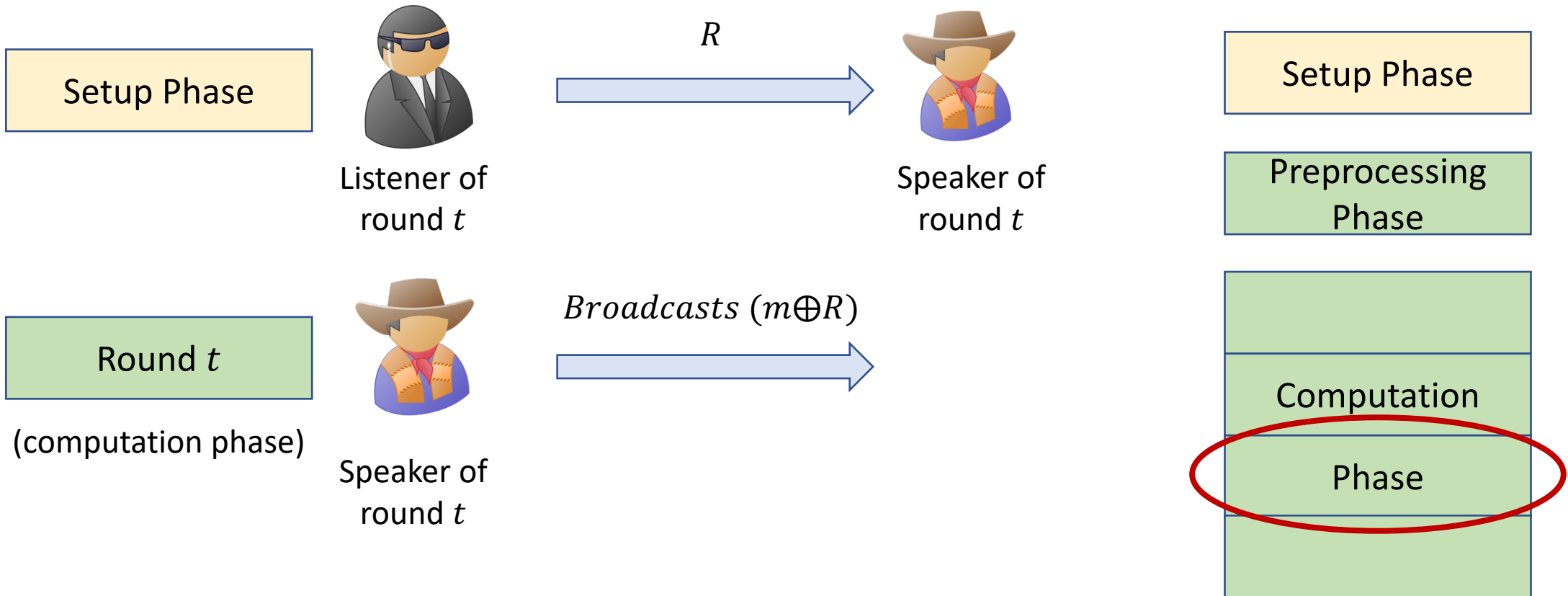
# Can we parallelize the first round with the setup phase?

## Conforming Protocol with setup



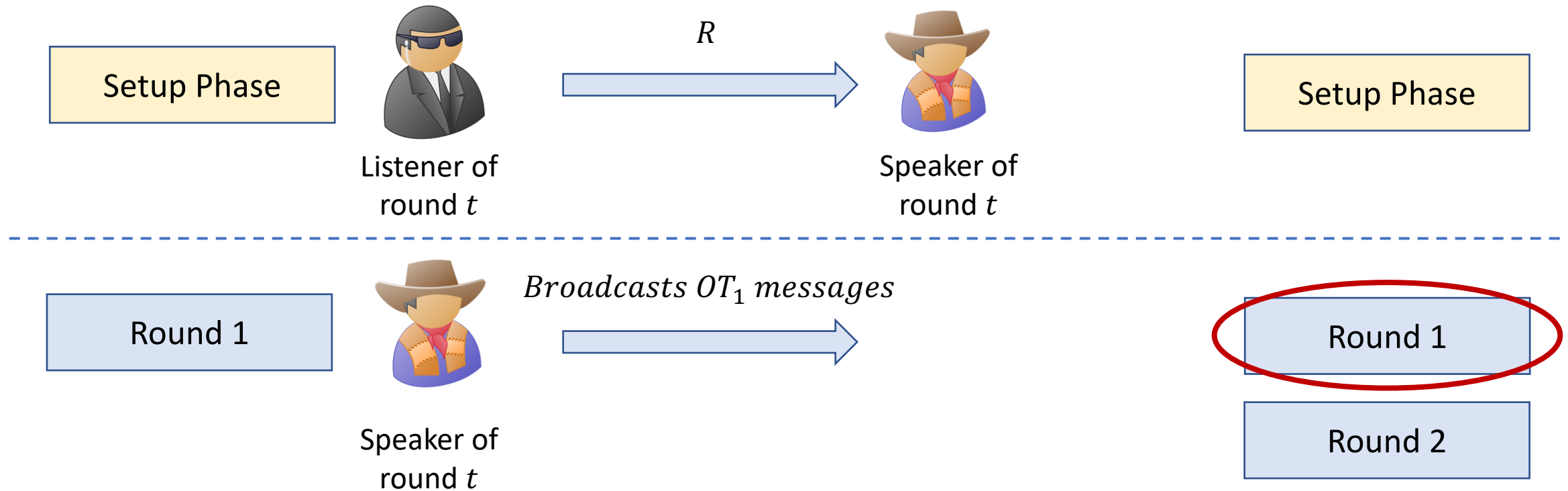
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## Conforming Protocol with setup



# Can we parallelize the first round with the setup phase?

## 2 Round Protocol with setup



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## 2 Round Protocol with setup

Setup Phase



Listener of  
round  $t$

$R$



Speaker of  
round  $t$

Round 1



Speaker of  
round  $t$

*Broadcasts  $OT_1$  messages*



$OT_1$  messages commit to all  
actions in the first round.

Can we parallelize the first round with the setup phase?

## 2 Round Protocol with setup

Setup Phase



Listener of  
round  $t$

$R$



Speaker of  
round  $t$

Round 1



Speaker of  
round  $t$

*Broadcasts  $OT_1$  messages*



$OT_1$  messages depend on  $R$   
which is not known before  
setup.



Can we parallelize the first round with the setup phase?

## 2 Round Protocol with setup

Setup Phase



Speaker of  
round  $t$

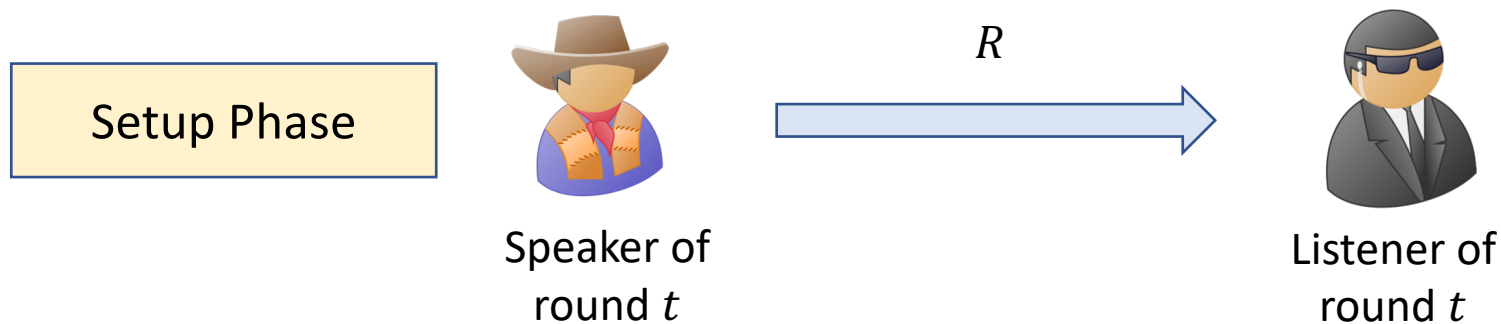
$R$



Listener of  
round  $t$

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## 2 Round Protocol with setup



- Similar problem arises.
- Transfers the problem to another round.

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## 2 Round Protocol with setup



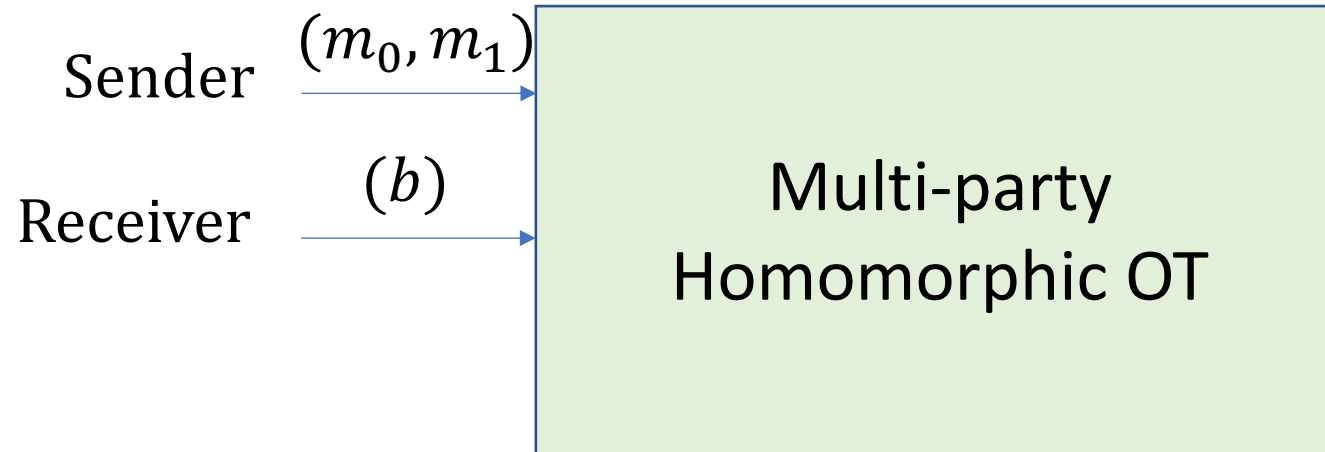
**This approach doesn't seem to work!**

- Similar problem arises.
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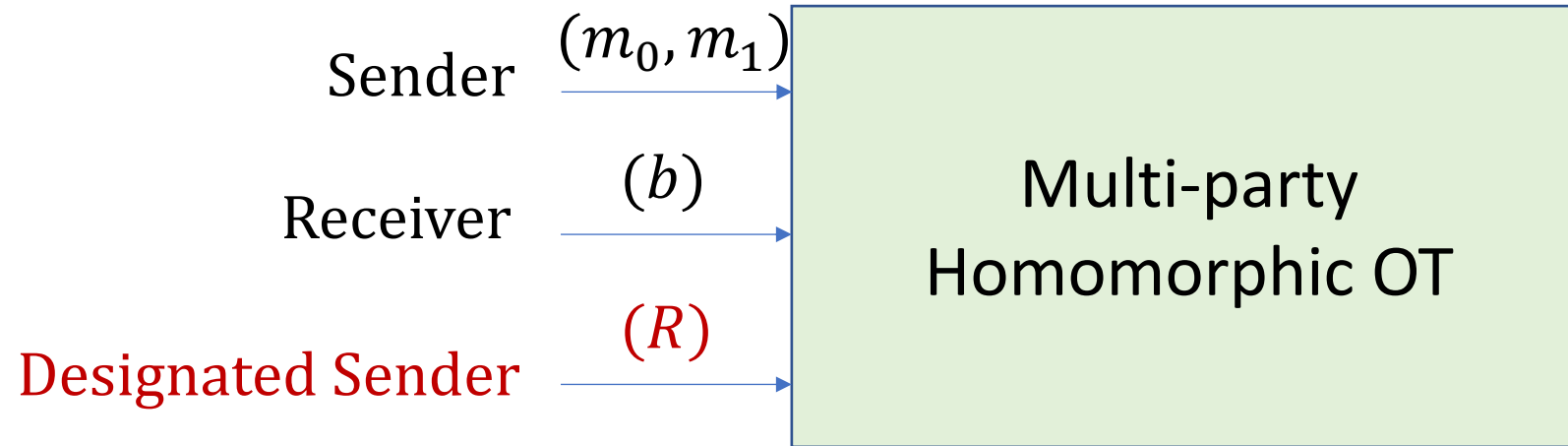
# Multi-party Homomorphic OT

- Multi-party protocol.
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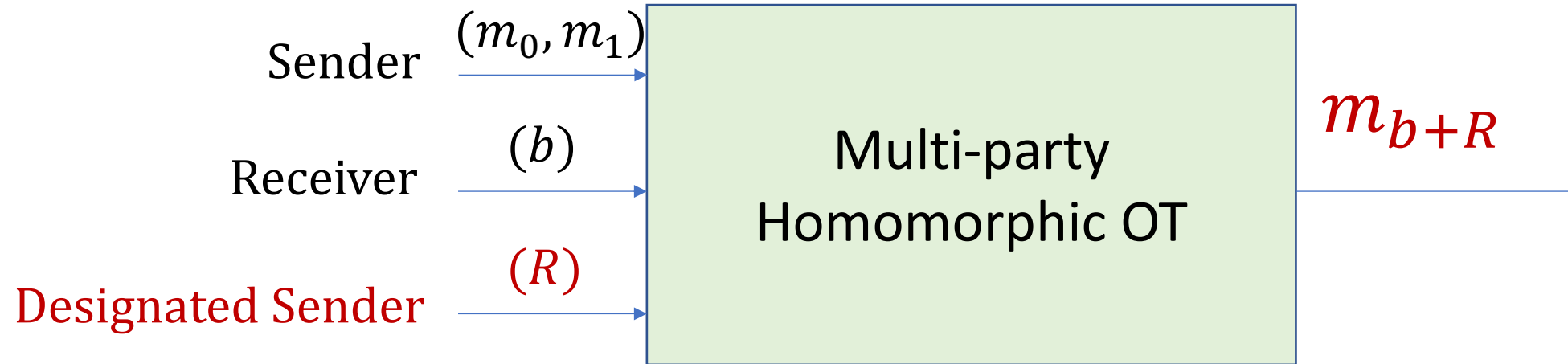
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# Multi-party Homomorphic OT

- The homomorphic OT functionality with sender inputs  $(m_0, m_1)$ , receiver input  $(b)$  and designated sender input  $(R)$  can be represented as degree 2 polynomial in  $\mathbb{F}_2$ .

$$m_{b+R} = m_0(1 + b + R) + m_1(b + R)$$



# Parallelizing using MHOT

## 2 Round Protocol with setup



Listener of round  $t$

$R$



Speaker of round  $t$

Setup Phase



Speaker of round  $t$

*Broadcasts  $OT_1$  messages*



Round 1

# Parallelizing using MHOT

## 2 Round Protocol with setup



Listener of round  $t$

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Speaker of round  $t$

Setup Phase



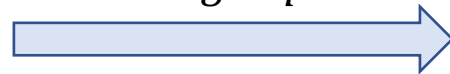
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Listener of round  $t$

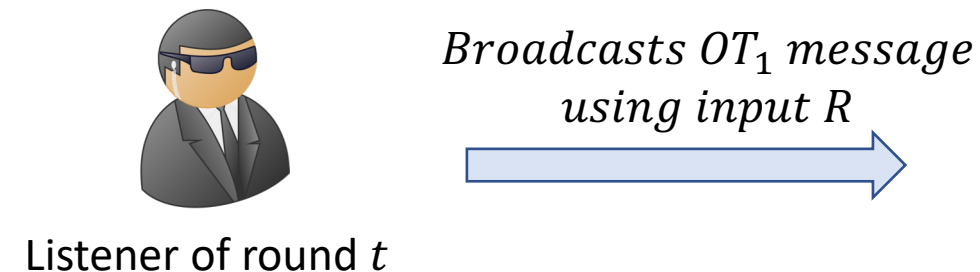
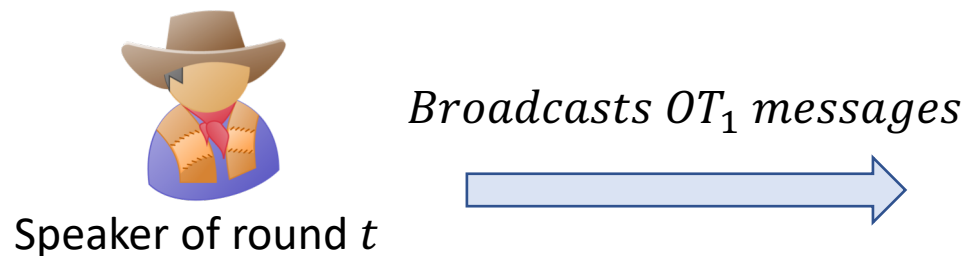
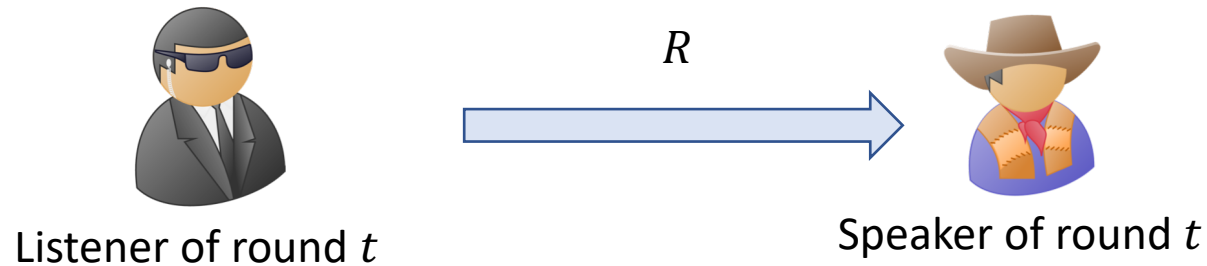
*Broadcasts  $OT_1$  message  
using input  $R$*



Round 1

# Parallelizing using MHOT

## 2 Round Protocol with setup parallelized



Setup Phase

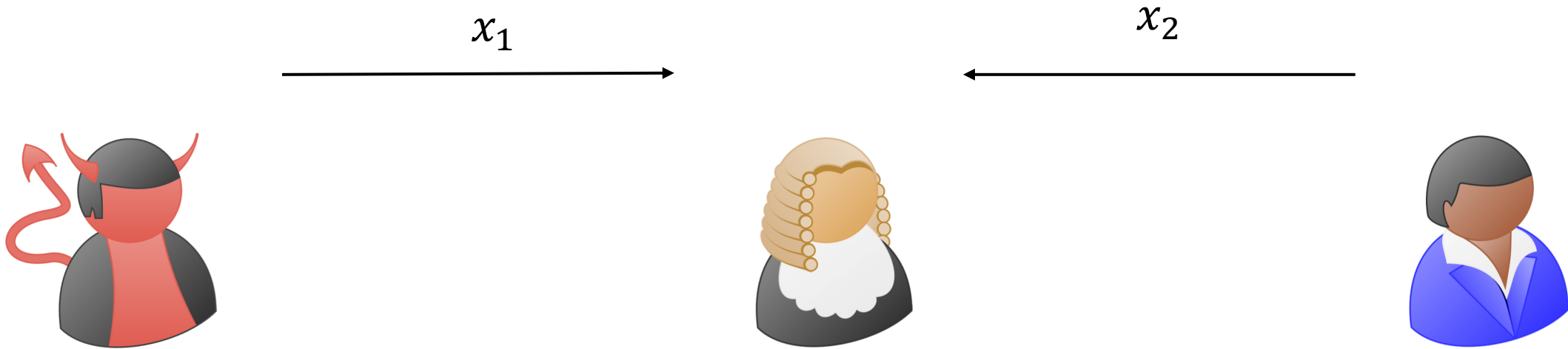
Round 1

The homomorphism property of the multi-party OT allows us to parallelize

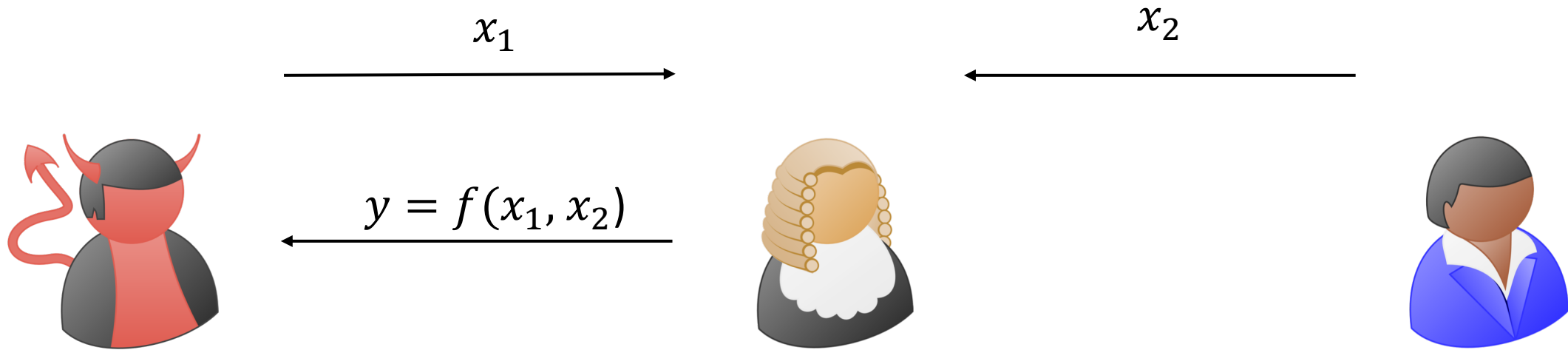
# Instantiating Multi-party Homomorphic OT

- [Ishai-Kushilevitz-Paskin10] give a construction for such a degree 2 polynomial computation protocol that satisfies statistical **t-privacy** with knowledge of outputs.

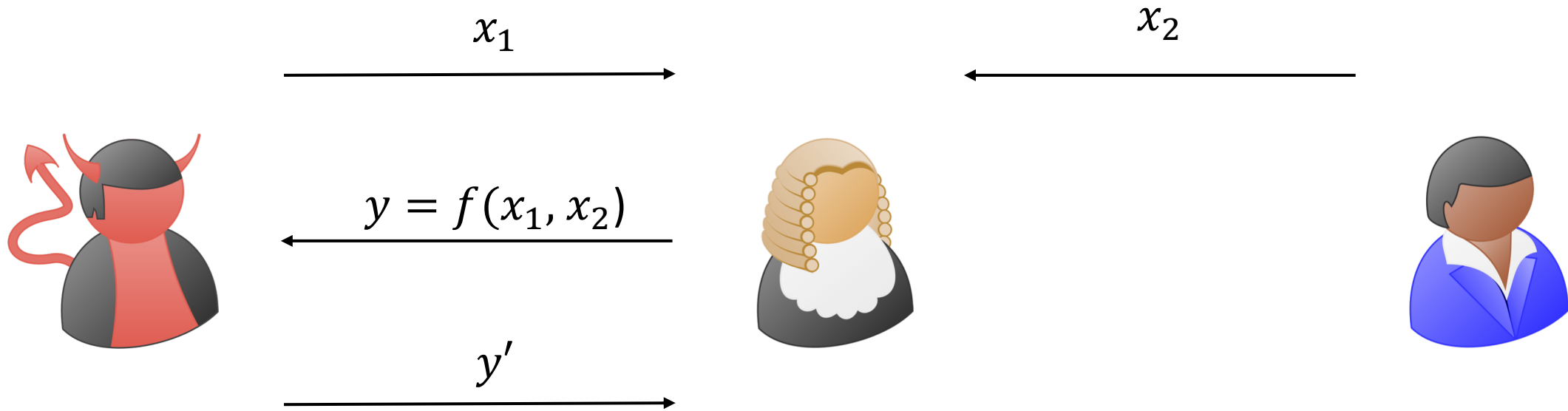
# Ideal World: Privacy with Knowledge of Outputs



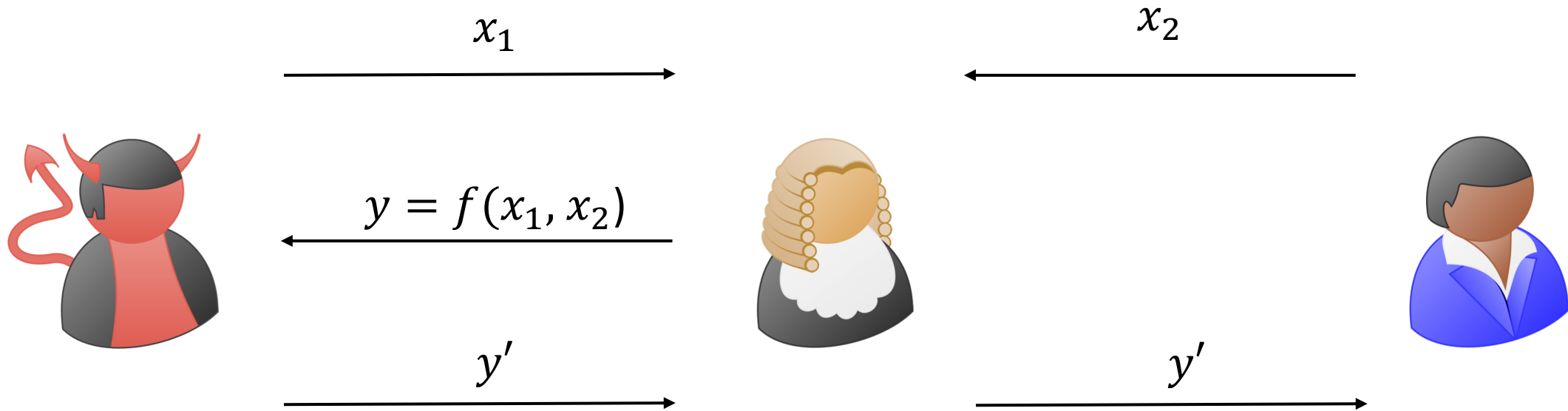
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# Instantiating Multi-party Homomorphic OT

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**Privacy with knowledge of outputs:** A weaker notion than security with abort that does not guarantee correctness of output of the honest parties.

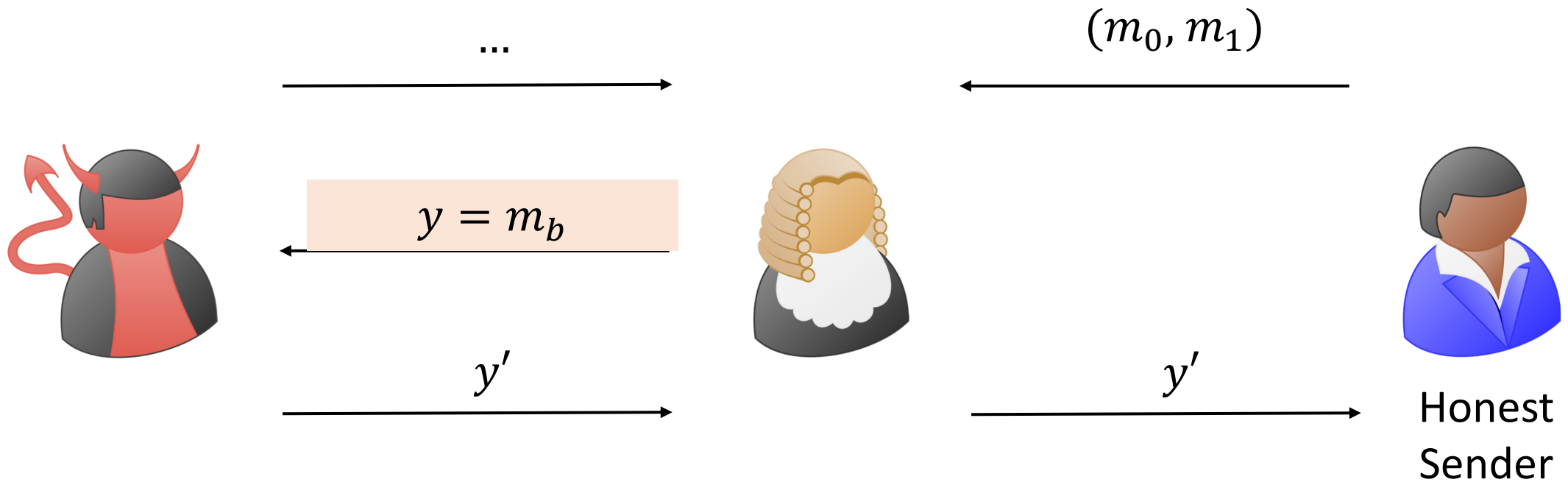
# Instantiating Multi-party Homomorphic OT

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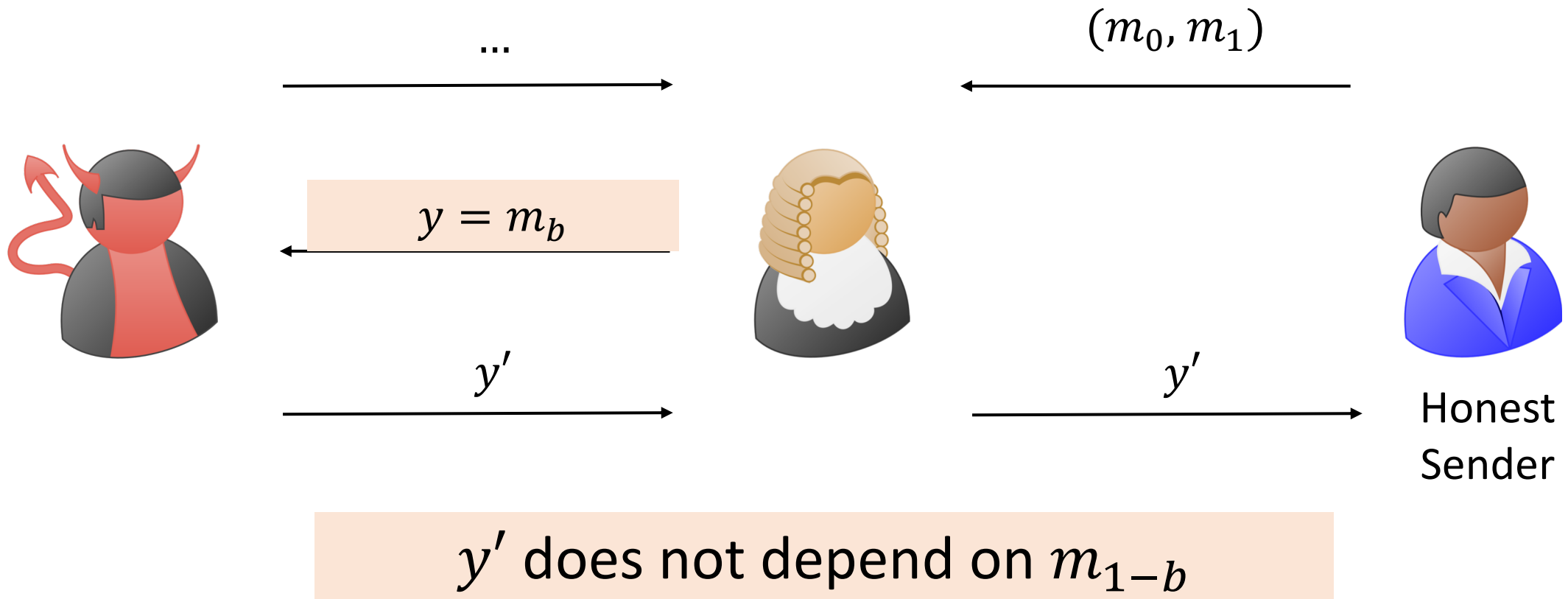
**Privacy with knowledge of outputs:** A weaker notion than security with abort that does not guarantee correctness of output of the honest parties.

**Challenge:** How to ensure correctness of honest party outputs?

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- OT functionality transmits wire labels for GC.
- Unless valid labels are transmitted, **GC remains private**.

<https://eprint.iacr.org/2018/572>

**Thank You.**

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