Secure Multiparty Computation with Free Branching

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Secure Multiparty Computation (MPC)



MPC protocol for computing $y = f(x_1, x_2, x_3, x_4, x_5)$

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Adversary learns nothing beyond the output y

MPC protocol for computing $y = f(x_1, x_2, x_3, x_4, x_5)$

Limitation of Existing Efficient MPC Protocols



Existing protocols rely on circuit representation of functions

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Communication complexity is linear in the size of circuit

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Existing protocols rely on circuit representation of functions

Communication complexity is linear in the size of circuit

What about functions that don't have an efficient circuit representation?



Circuit Representation:

depends on ALL 3 branches









Naïve use of existing MPC protocols will result in communication proportional to all 3 branches

Main Question

Circuit Representation: de

depends on ALL 3 branches

Can we design efficient MPC protocols for computing conditional branches, where communication only depends on the size of a single branch?



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Naïve use of existing MPC protocols will result in communication proportional to all 3 branches

MPC with Free Branching: Applications

Control flow instructions in computer programs

Collection of servers providing services that clients can pay and for and obliviously avail

k = # branches

|C| = size of largest branch

| Result | No. of Parties | Communication | Security | Rounds | Type of Circuits |
|---------------|-------------------|-------------------------|-------------|-----------------|------------------|
| [HK20] | 2 | <i>O</i> (<i>C</i>) | Semi-Honest | Non-interactive | Boolean |
| [HK21] | 2 | <i>O</i> (<i>C</i>) | Semi-Honest | Non-interactive | Boolean |
| [HKP20,HKP21] | n | $O(kn^2 C)$ | Semi-Honest | Linear in depth | Boolean |

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No *n*-party protocol where communication only depends on the size of one branch

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No maliciously secure protocol

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No protocol for arithmetic circuits

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| Our Work | n | $O(n^2 \mathcal{C})$ | Semi-Honest | Linear in depth | Arithmetic |

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| Our Work | n | $O(n^2 \mathcal{C})$ | Semi-Honest | Linear in depth | Arithmetic |
| Our Work | n | $O(n^2s \mathcal{C})$ | Malicious | Linear in depth | Arithmetic |

Statistical security parameter

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| Our Work | n | $O(n^2s C)$ | Malicious | Linear in depth | Arithmetic |
| Our Work | n | $O(n^2 C)$ | Semi-Honest | Constant | Boolean |

k = # branches

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Main Ideas





Multiplexer



Multiplexer







Our Work: High-Level Approach



Obliviously select the active branch and only evaluate that on correct inputs

Our Work: High-Level Approach



Obliviously select the active branch and only evaluate that on correct inputs

Since the active branch must remain hidden, how does one compute on a hidden function?

Our Initial Observation: Similarities with PFE



Private Function Evaluation only 1 person knows the function!

Our Initial Observation: Similarities with PFE

$f\in\{f_1,f_2,f_3\}$



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MPC for Conditional Branches No one knows the function!

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$f\in\{f_1,f_2,f_3\}$



Private Function Evaluation only 1 person knows the function!



MPC for Conditional Branches No one knows the function!

Parties collectively hold information about the active branch

Talk Outline

Overview of the [MS13] PFE protocol

Our semi-honest non-constant round protocol for conditional branches

Performance Evaluation

Remarks on additional results
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PFE: How to Hide Circuit Topology? [MS13]

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Hide wire configurations

PFE: How to Hide Circuit Topology? [MS13]





Hide wire configurations

Hide gate functions







For every gate g:



- Let $type_g = 0$: if g is a multiplication gate
- Let $type_g = 1$: if g is an addition gate



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Given shares $[L_g]$, $[R_g]$ of left and right input wires, compute



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Compute using an MPC that can operate over shares!

Given shares $[L_g]$, $[R_g]$ of left and right input wires, compute $[type_g]$. $([L_g], [R_g]) + (1 - [type_g])$. $([L_g] + [R_g])$













 x_4





 x_4

 x_3





 x_4

 x_3





 x_1

 x_2

 x_4

2























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Conditional Branches: Hiding Gate Functions



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Conditional Branches: Hiding Gate Functions



Let's assume parties have secret sharing of unary representation of α , i.e., $[b_1], \dots, [b_k]$

For every gate *g*:

$$[type_g] = [b_1].type_{1,g} + \dots + [b_k].type_{k,g}$$

Given shares $[L_g]$, $[R_g]$ of left and right input wires, compute $[type_g]$. $([L_g], [R_g]) + (1 - [type_g])$. $([L_g] + [R_g])$













 $[omask_{\pi_{\alpha}(w)}] = [b_1][omask_{\pi_1(w)}] + \dots + [b_1][omask_{\pi_1(w)}]$






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Using Threshold Linearly Homomorphic Encryption!

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Using Threshold Linearly Homomorphic Encryption!

Encrypt b_1, \dots, b_k :

$$< b_1 > \stackrel{Enc}{\leftarrow} [b_1], \dots, < b_k > \stackrel{Enc}{\leftarrow} [b_k]$$

 $[omask_{\pi_{\alpha}(w)}] = [b_1][omask_{\pi_1(w)}] + \dots + [b_1][omask_{\pi_1(w)}]$

Using Threshold Linearly Homomorphic Encryption!

Encrypt
$$b_1, \dots, b_k$$
:

$$\begin{array}{c} \langle b_1 \rangle \stackrel{Enc}{\leftarrow} [b_1], \dots, \langle b_k \rangle \stackrel{Enc}{\leftarrow} [b_k] \end{array} \end{array}$$
Each party p computes for every wire w :

$$\begin{array}{c} \langle omask_{\pi_{\alpha}(w)}^{(p)} \rangle = \langle b_1 \rangle . \ [omask_{\pi_1(w)}]^{(p)} + \dots + \langle b_k \rangle . \ [omask_{\pi_k(w)}]^{(p)} \end{array}$$

 $[omask_{\pi_{\alpha}(w)}] = [b_1][omask_{\pi_1(w)}] + \dots + [b_1][omask_{\pi_1(w)}]$

Using Threshold Linearly Homomorphic Encryption!

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for every wire w : $< omask_{\pi_{\alpha}(w)}^{(p)} > = < b_1 > [omask_{\pi_1(w)}]^{(p)} + \dots + < b_k > [omask_{\pi_k(w)}]^{(p)}$ Aggregate: $< omask_{\pi_{\alpha}(w)} > = \sum_p < omask_{\pi_{\alpha}(w)}^{(p)} >$

 $[omask_{\pi_{\alpha}(w)}] = [b_1][omask_{\pi_1(w)}] + \dots + [b_1][omask_{\pi_1(w)}]$

Using Threshold Linearly Homomorphic Encryption!



Communication only depends on the size of one branch







Online Phase x_1 x_4 x_2 χ_3 For every gate *g*: After computing *g* Compute $[u_c] = [z_c] + [omask_c]$ Z_{C} and Reconstruct u_c С Gate *g* Incoming label а b $\pi(a)$ $\pi(b)$ Outgoing label Z_a Actual wire values Z_b







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We use an implementation of semi-honest MASCOT from the MP-SPDZ library

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3 parties, 2¹⁶ gates in each branch

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We implement the [CDN01] Protocol

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3 parties, 2¹⁶ gates in each branch

We implement the [CDN01] Protocol Comm: CDN Branching Time: CDN Branching 4,000 Comm: CDN Parallel Time: CDN Paralle 350 3,500 3,000 (su 2,500 all 2,000 1,500 1,000 50 500 0 0 26 2^1 2² 2³ 24 25 21 2² 2³ 24 25 26 Branches Branches Communication **Run-Time**

3 parties, 2¹⁶ gates in each branch

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This protocol can be extended to malicious security

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Incurs additional multiplicative overhead dependent on the statistical security parameter

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Naïve multi-party garbling using the above approach results in nonblack box use of cryptography

We present an alternate solution using the linearly key-homomorphic PRFs based garbling approach from [BLO17]

Conclusion

A multi-party protocol for securely computing conditional branches, where the total communication only depends on the size of the largest branch

Extensions to malicious security and a semi-honest constant round protocol

Thank You!