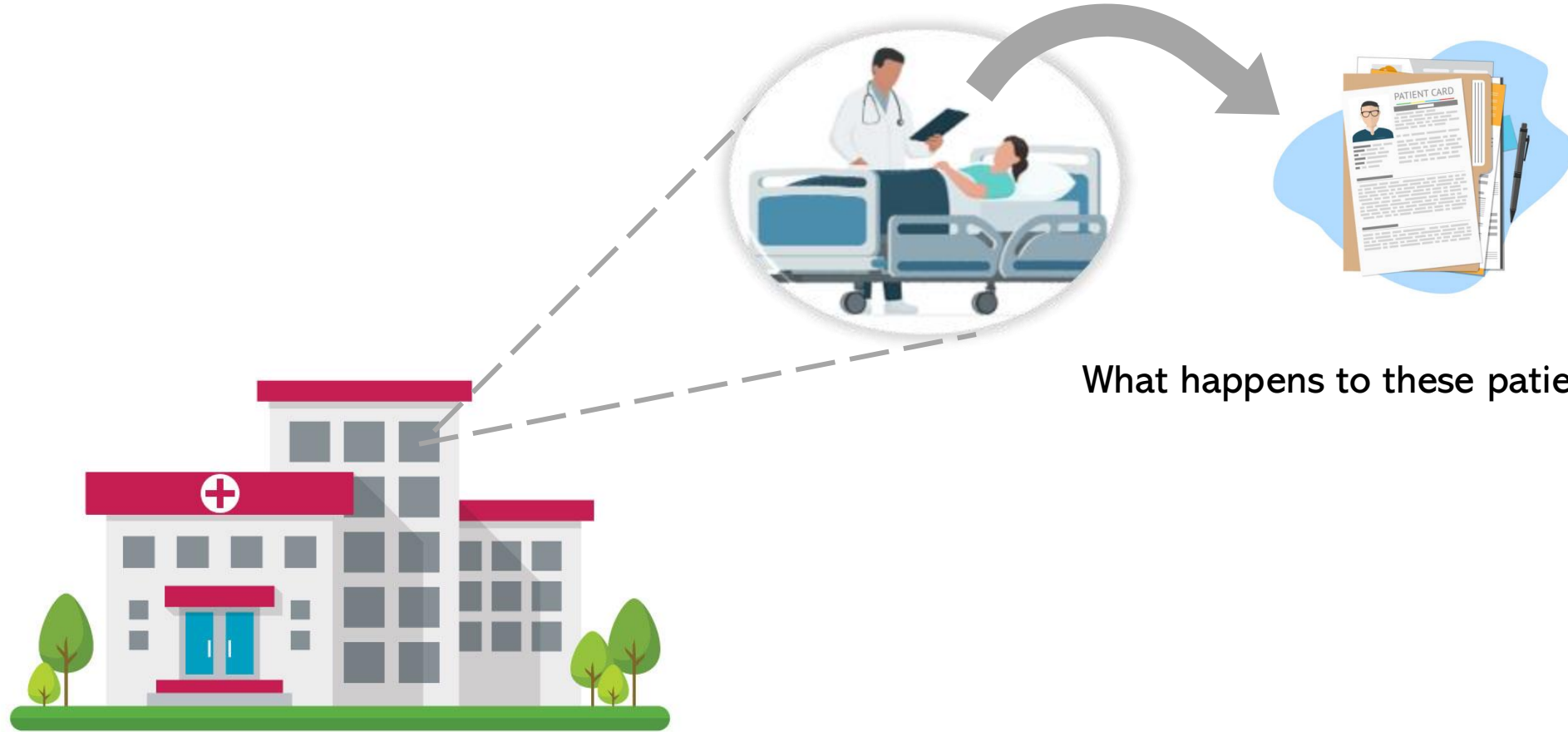


Private and Verifiable Delegation of Computation

Aarushi Goel

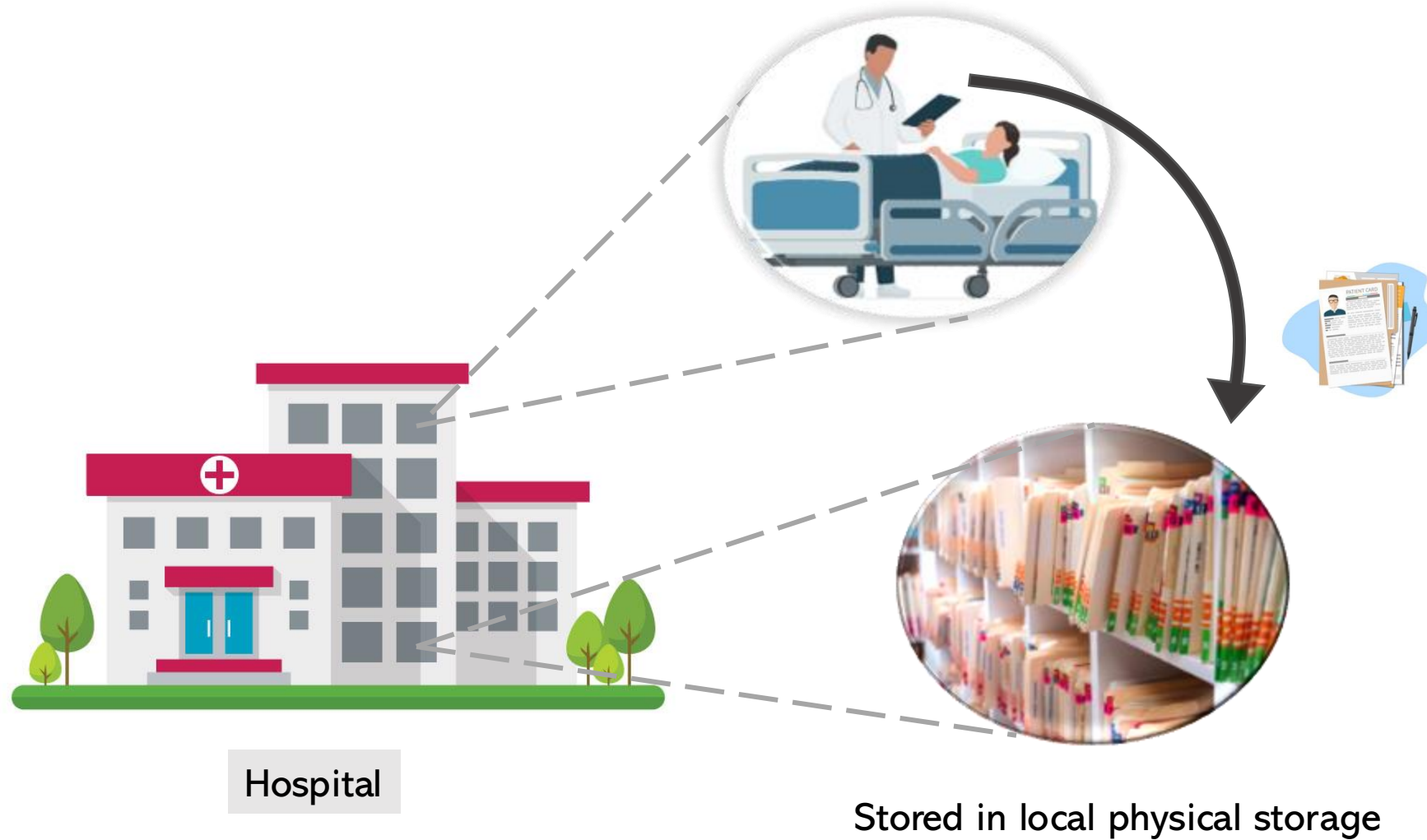




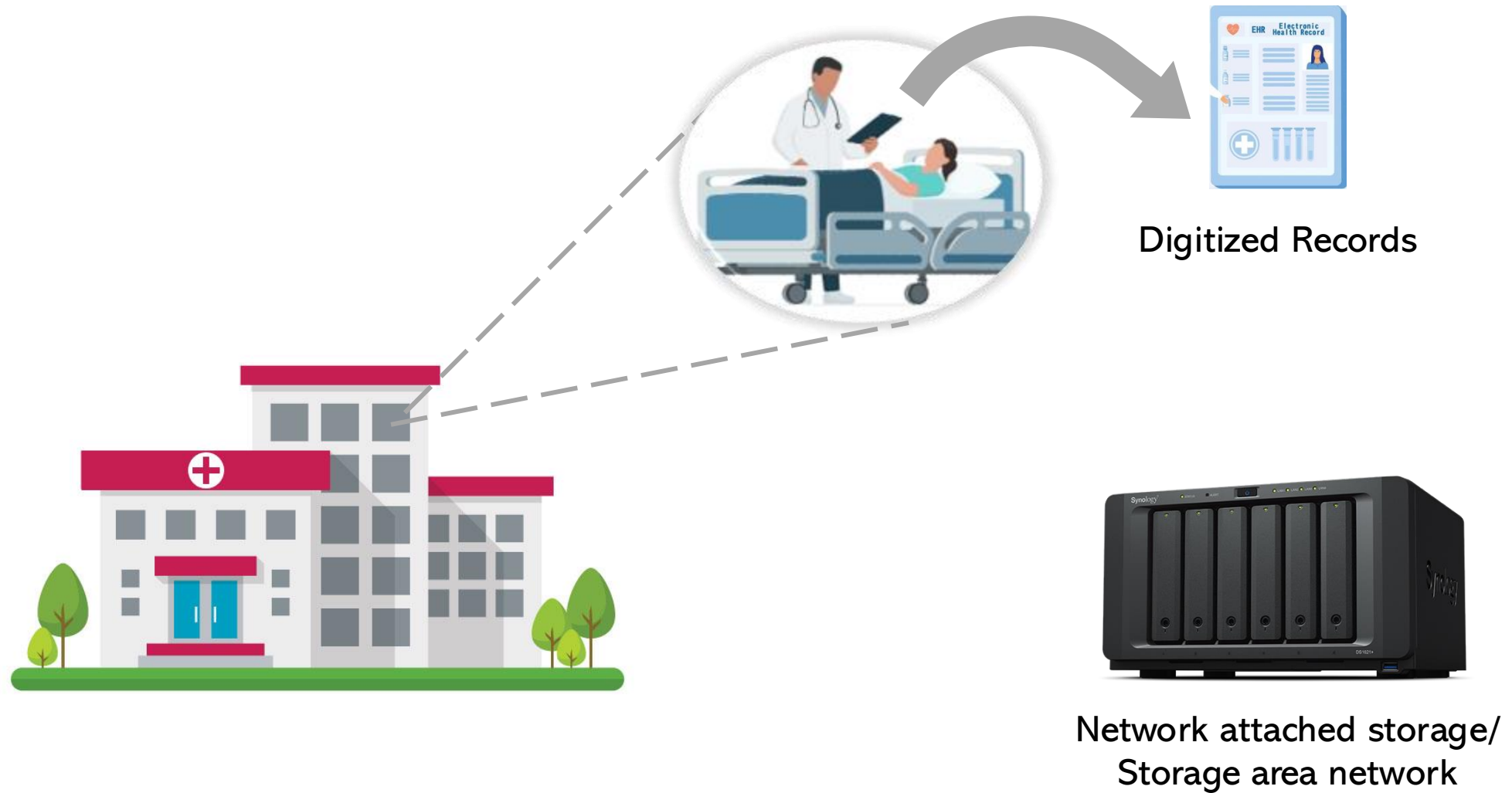
What happens to these patient records?

Hospital

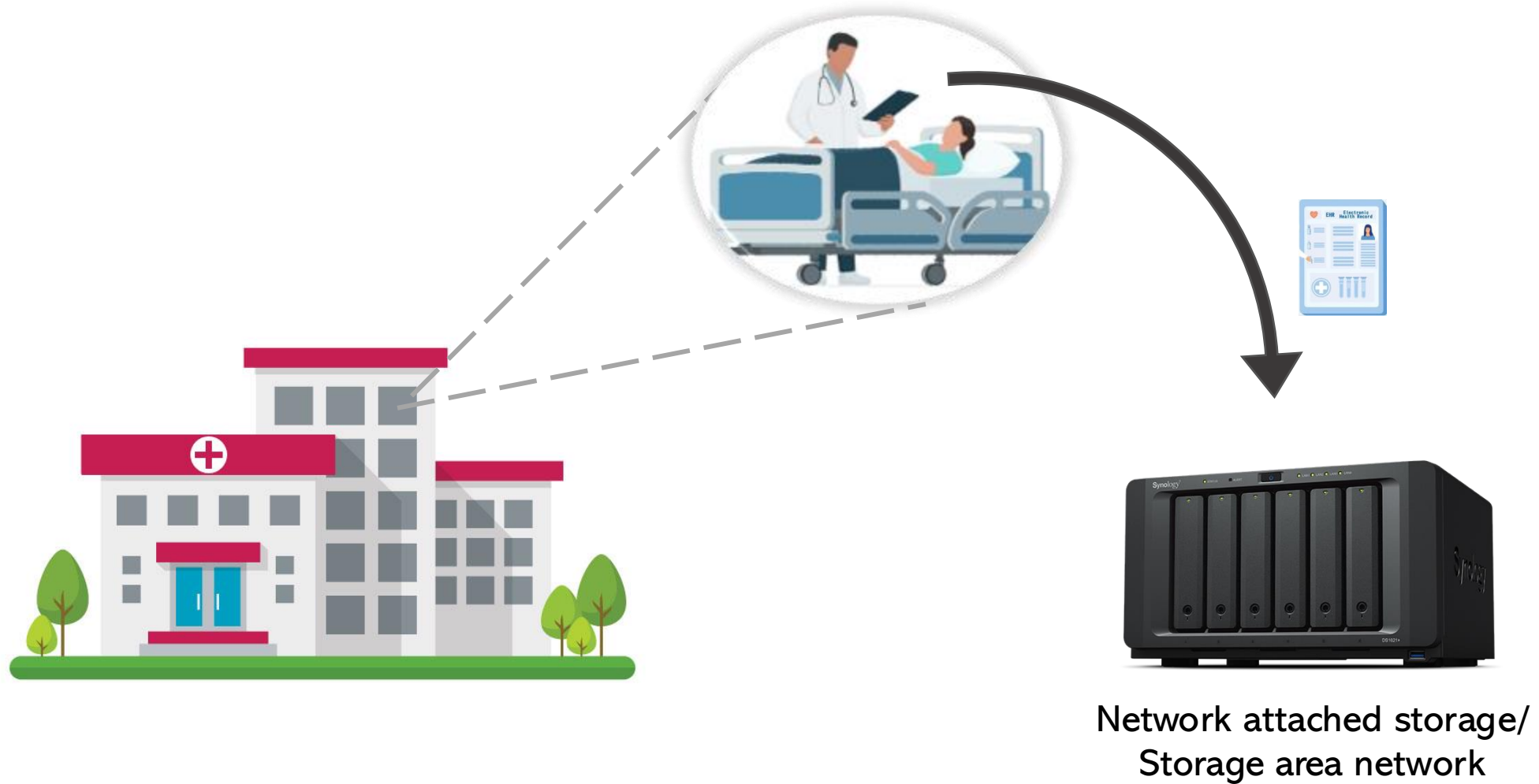
Until Late 1980s



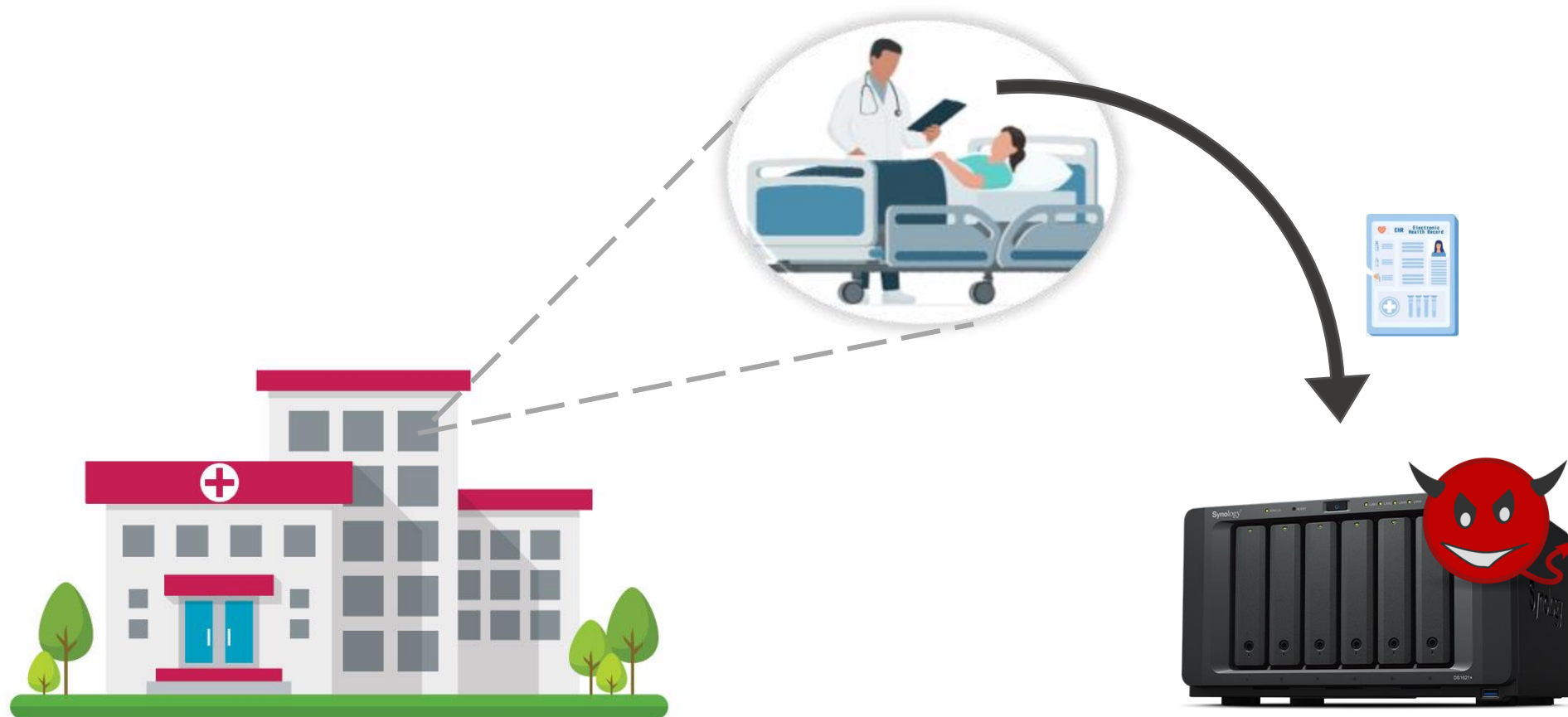
Late 1980s – Late 2000s



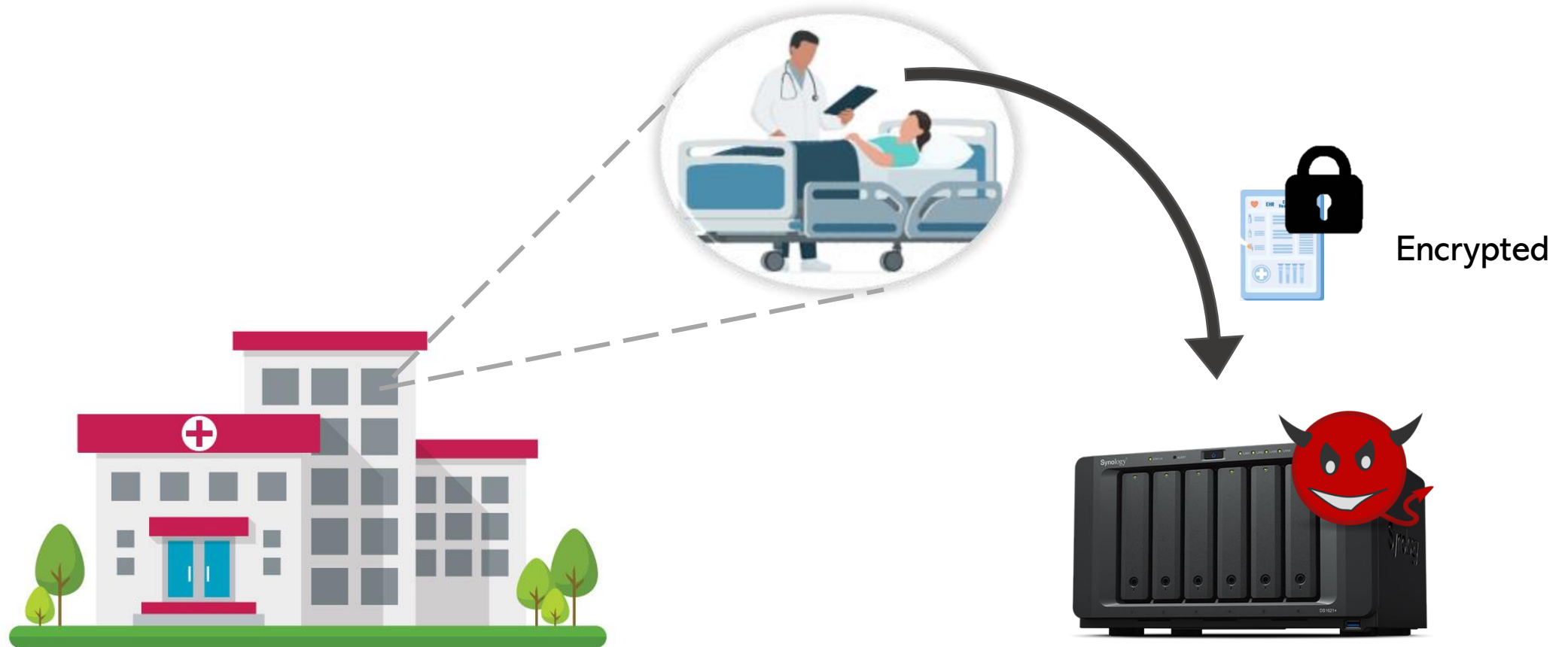
Late 1980s – Late 2000s



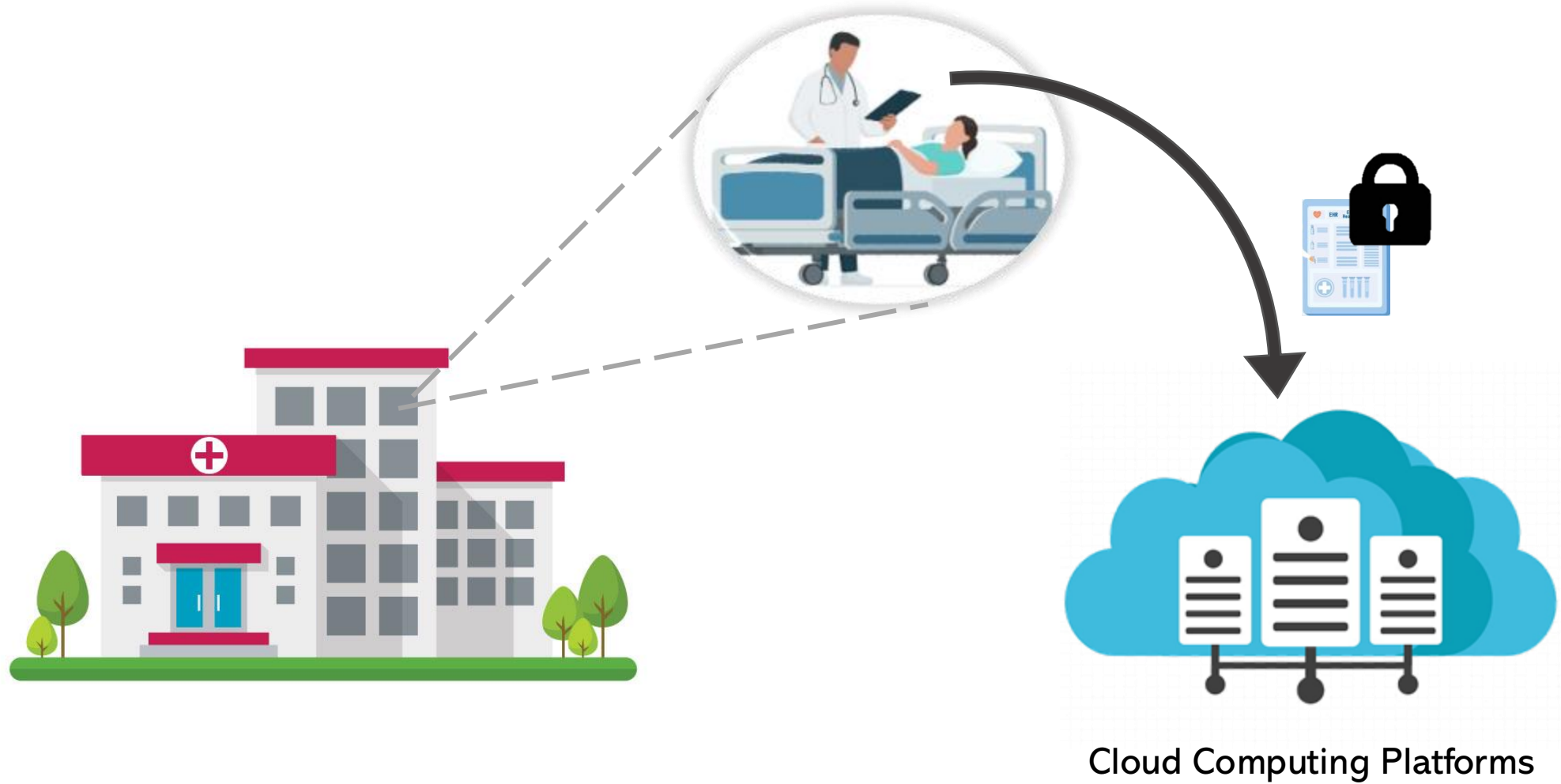
Late 1980s – Late 2000s



Late 1980s – Late 2000s



Since Late 2000s



Since Late 2000s

Search
encrypted database



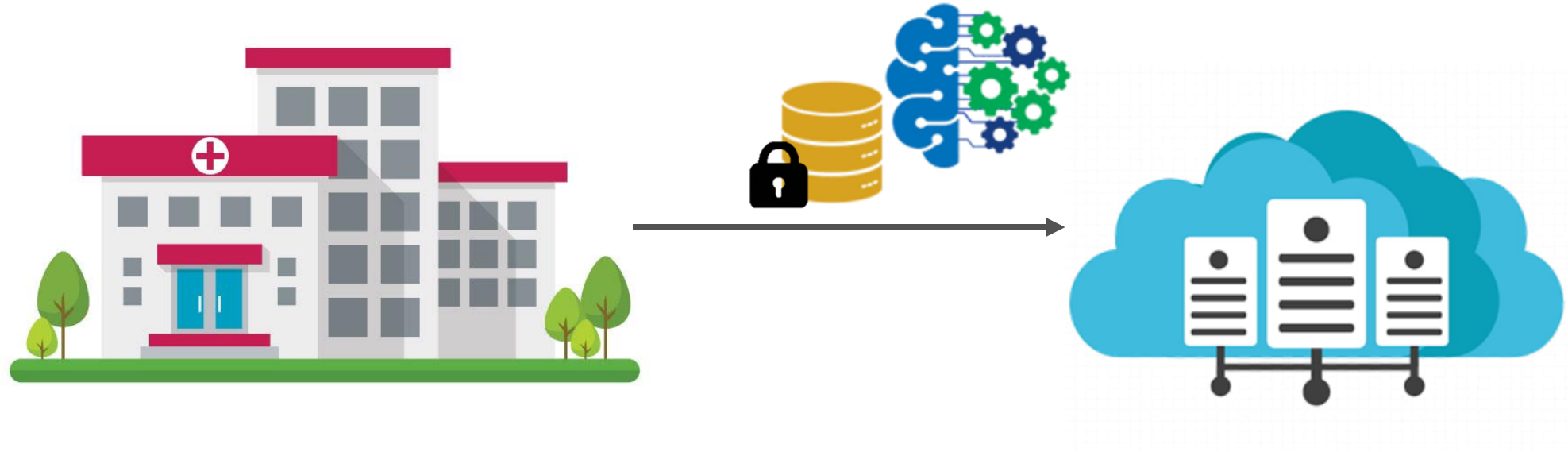
Since Late 2000s

Statistical analysis on
encrypted database



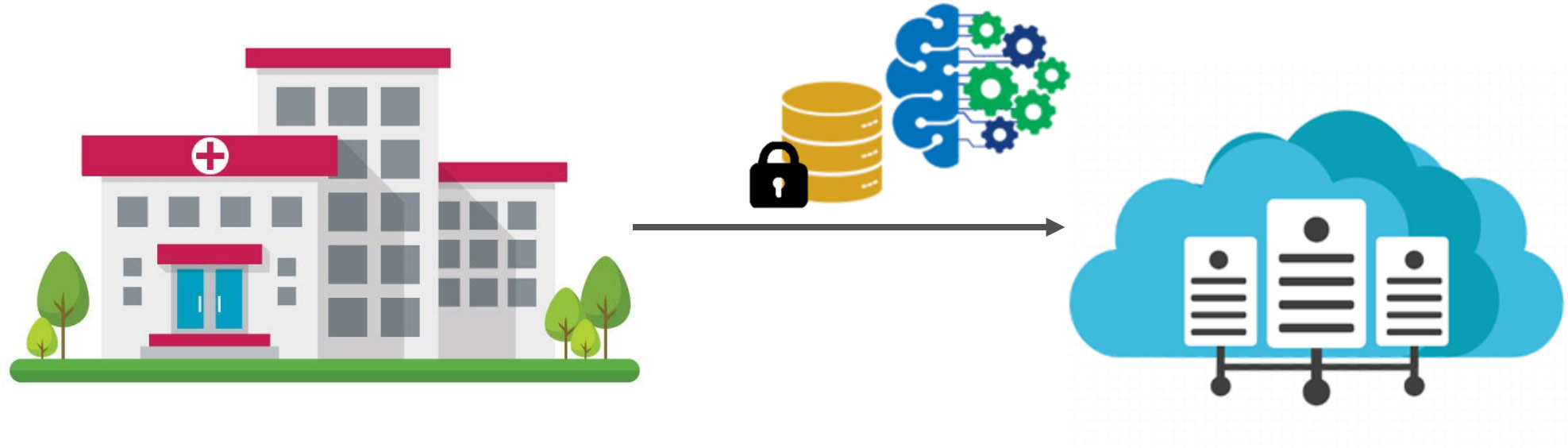
Modern Applications

Train an ML model
encrypted database



Since Late 2000s

How does one compute on encrypted data?



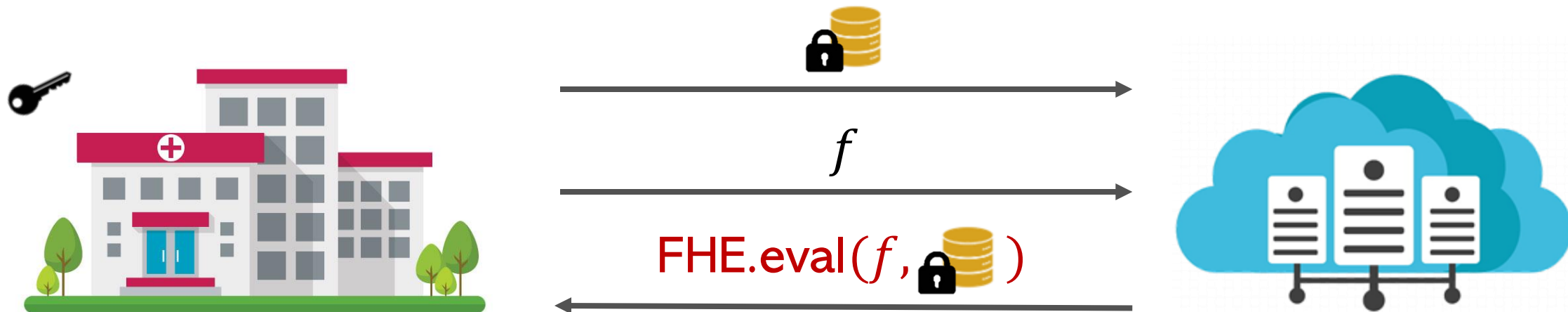
Fully Homomorphic Encryption

[Gentry09]



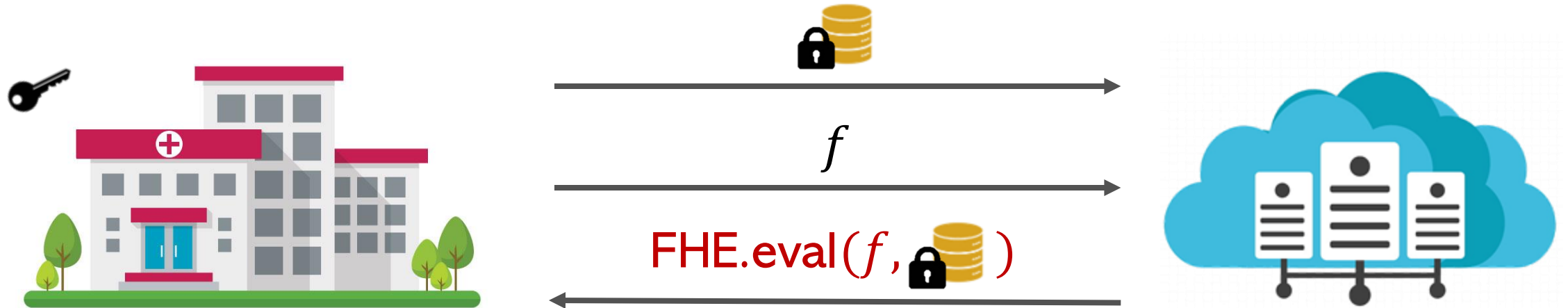
Fully Homomorphic Encryption

[Gentry09]



Fully Homomorphic Encryption

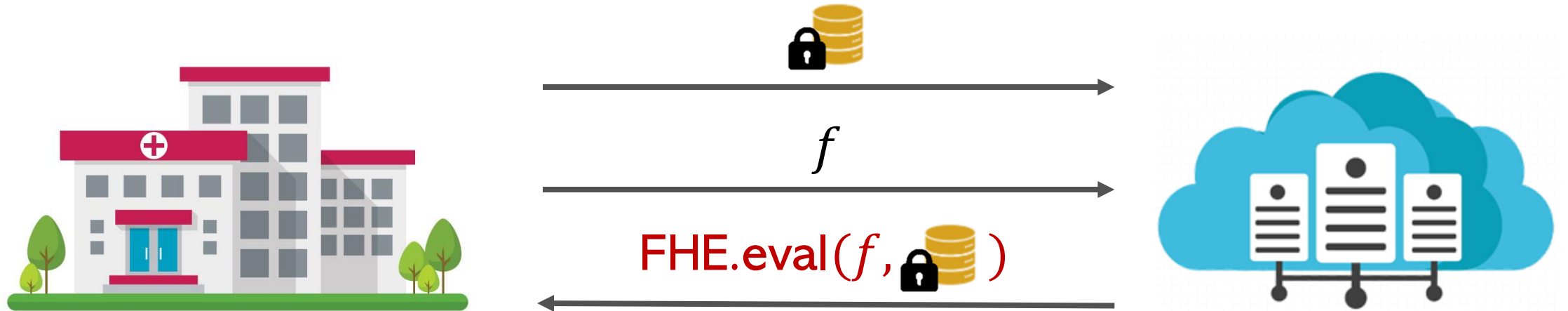
[Gentry09]



$\text{FHE.eval}(f, \text{[padlock icon]})$

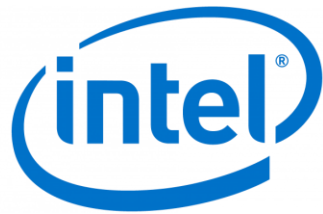
Fully Homomorphic Encryption

[Gentry09]

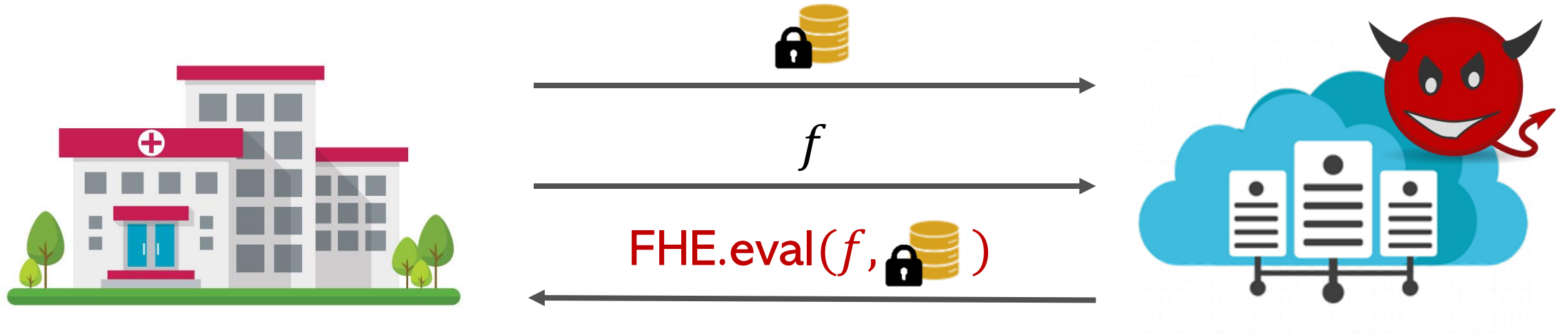


$$\text{FHE.eval}(f, \text{padlock, coins}) \xrightarrow{\text{key}} f(\text{coins})$$

FHE in Industry

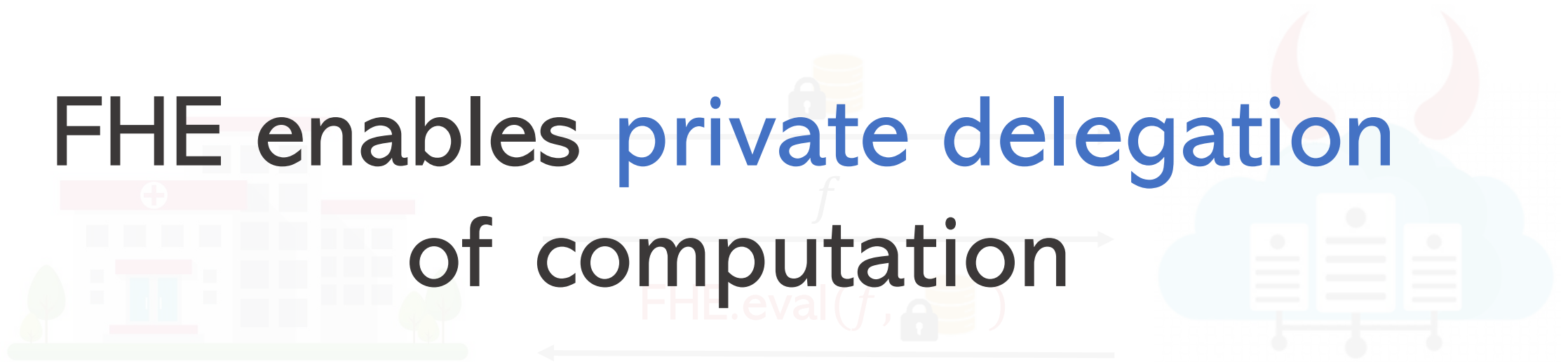


Can we trust the cloud to compute $\text{FHE.eval}(f, \text{data})$ honestly?



Can we trust the cloud to compute $\text{FHE.eval}(f, \text{data})$ honestly?

FHE enables **private delegation**
of computation



Main Question:

Can we enable **private** **and verifiable**
delegation of computation?

Main Question:

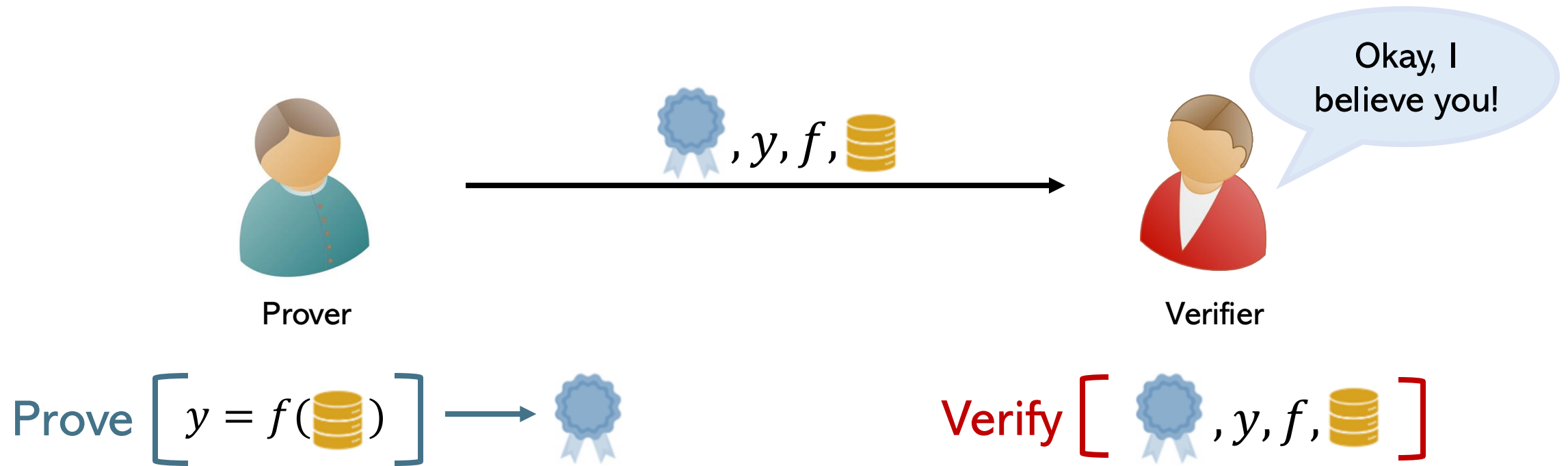
Can we enable **private** **and** **verifiable**
delegation of computation?

FHE

??

Verifiable Computation

Succinct Proofs/SNARKs [Kil92, Mic94]



SNARKs in Industry

Action1 raises \$20M to implement zero-knowledge architecture into its platform

RISC Zero raises \$40M in new funding for blockchain effort

Zero-knowledge proof startup zCloak Network raises \$5.8 million



Zero-Knowledge Proof Startup Proven Raises \$15.8M

Zero-Knowledge Privacy Startup Webb Protocol Raises \$7M

Ingonyama Raises \$21 Million for Zero-Knowledge Proof Acceleration and Semiconductor Development



Zcash



NEXUS



ETHERIUM



Can we enable **private** **and** **verifiable**
delegation of computation?

FHE

SNARKs

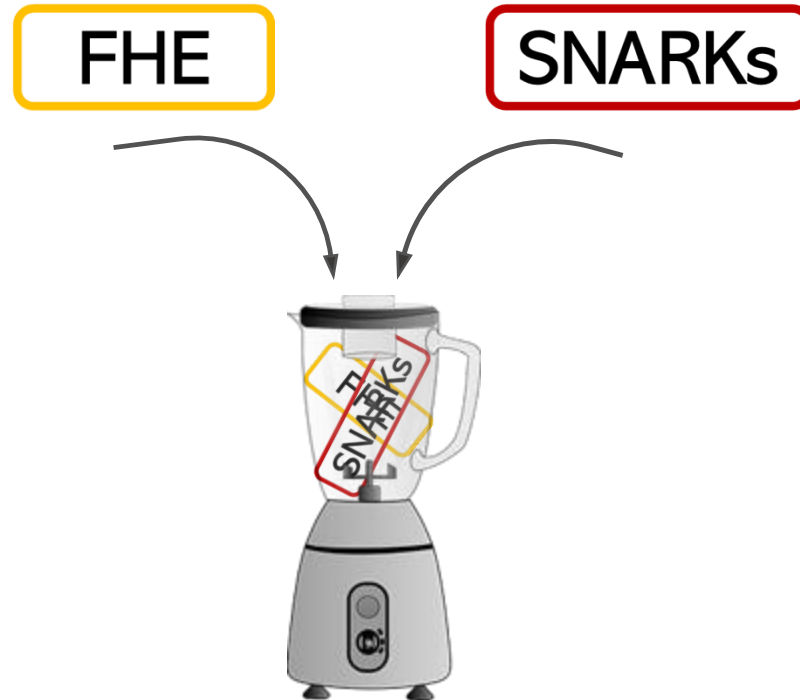
Can we enable **private** **and verifiable**
delegation of computation?

FHE

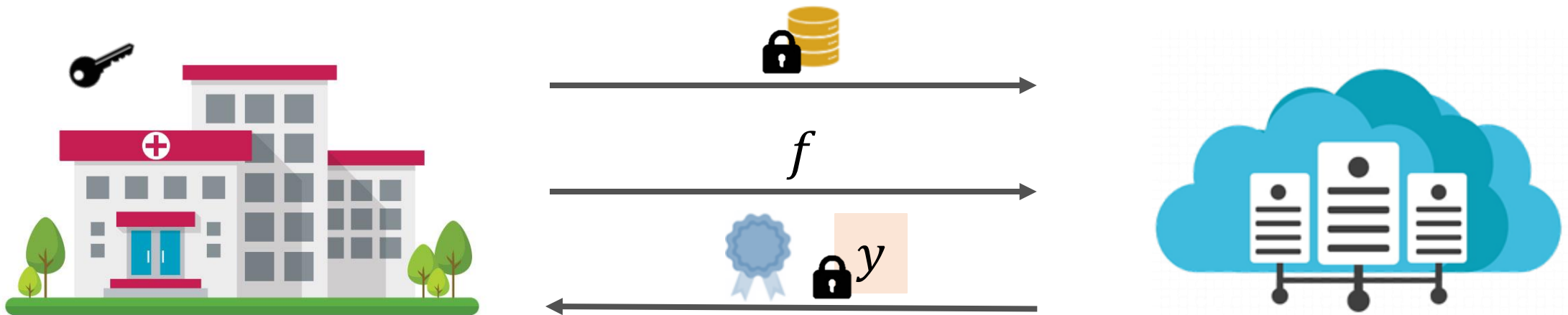
SNARKs



Can we enable **private** **and verifiable**
delegation of computation?



FHE + SNARKs: Strawman Approach



Verify $\left[\text{seal}, \text{lock } y, f, \text{lock coins} \right]$

$\text{lock } y \xrightarrow{\text{key}} y = f(\text{coins})$

$\text{lock } y = \text{FHE.eval}(f, \text{lock coins})$

Prove $\left[\text{lock } y = \text{FHE.eval}(f, \text{lock coins}) \right] \rightarrow \text{seal}$

These Technologies Have **Overheads!**

$$\text{Time} \left[\text{Prove} \left[y = f(\text{DB}) \right] \right] \gg a \times \text{Time} \left[f(\text{DB}) \right]$$

$$\text{Time} \left[\text{FHE.eval}(f, \text{DB} \text{ with lock}) \right] \gg b \times \text{Time} \left[f(\text{DB}) \right]$$

FHE + SNARKs: Strawman Approach

$$\text{Time} \left[\text{Prove} \left[y = \text{FHE.eval}(f, \text{lock}(\text{data})) \right] \right] \gg ab \times \text{Time} \left[f(\text{data}) \right]$$

Prohibitively slow!

Our Result

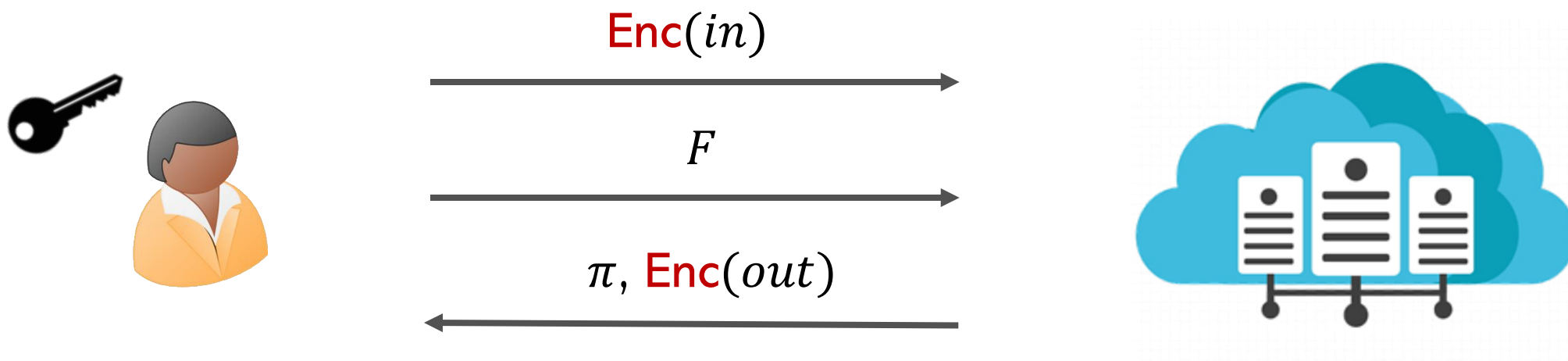
[GGW | CRYPTO24]

An efficient way to combine **FHE + SNARKs** for **private and verifiable delegation of computation** to a **single untrusted server**.

$$\text{Server's RunTime} \approx (a + b) \times \text{Time} \left[f(\text{💰}) \right]$$

Only makes black-box use of FHE.

FHE + SNARKs: Strawman Approach



Verify $\left[\pi, \text{Enc}(out), \text{Enc}(in), \text{FHE.eval}(F, \cdot) \right]$

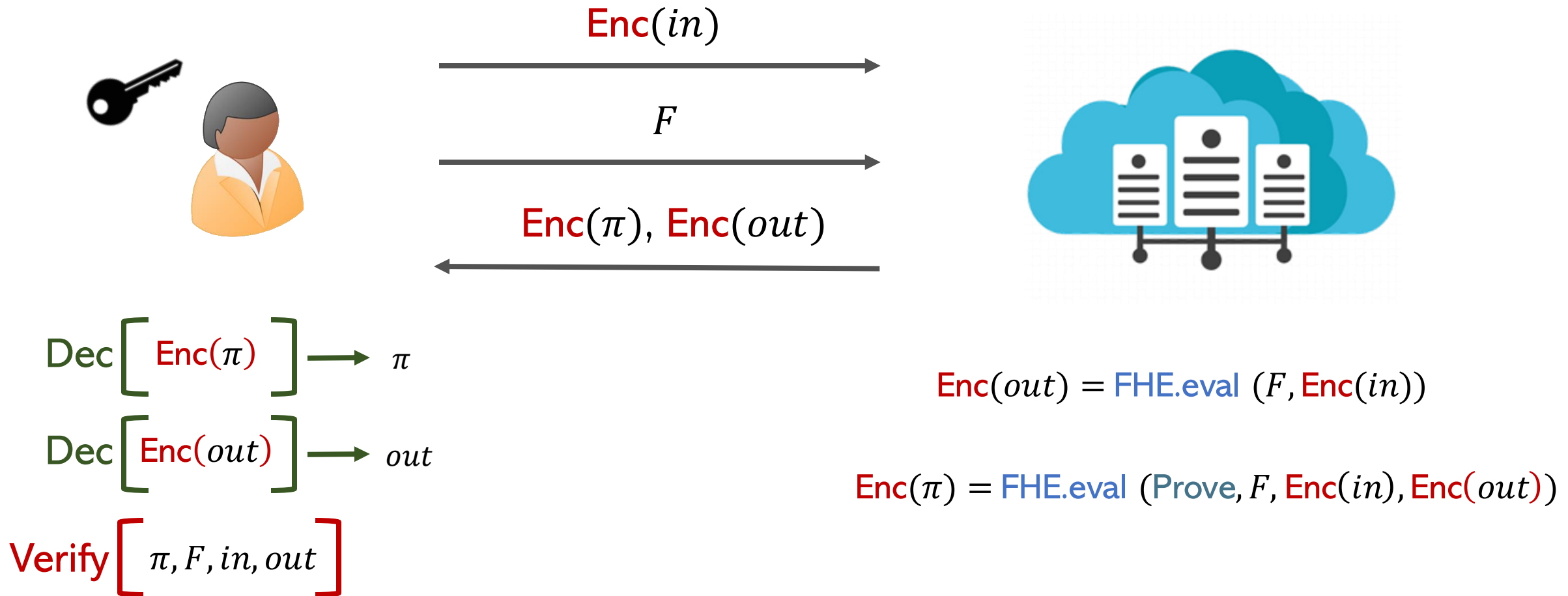
Dec $\left[\text{Enc}(out) \right] \rightarrow out$

$\text{Enc}(out) = \text{FHE.eval}(F, \text{Enc}(in))$

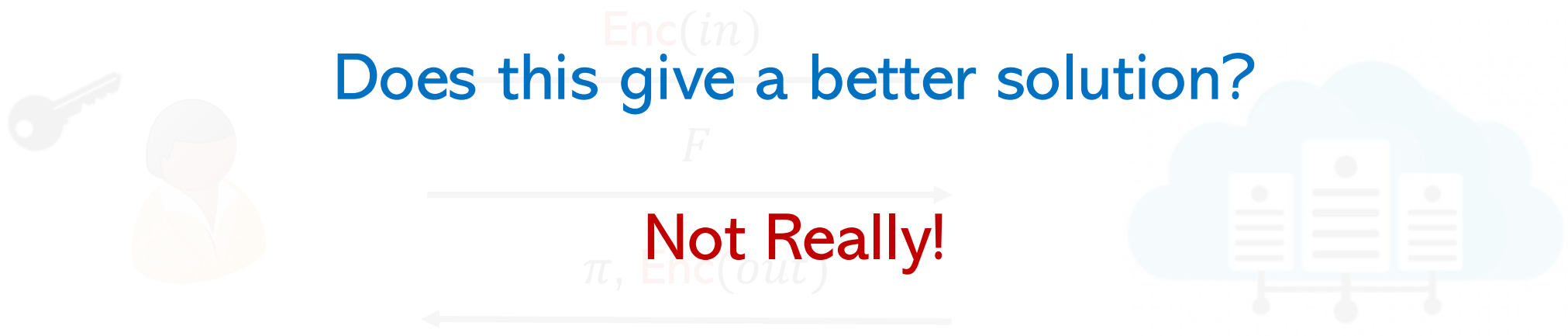
Prove $\left[\text{Enc}(out) = \text{FHE.eval}(F, \text{Enc}(in)) \right] \rightarrow \pi$

FHE + SNARKs: Another Idea?

Compute an **encrypted** proof



FHE + SNARKs: Another Idea?

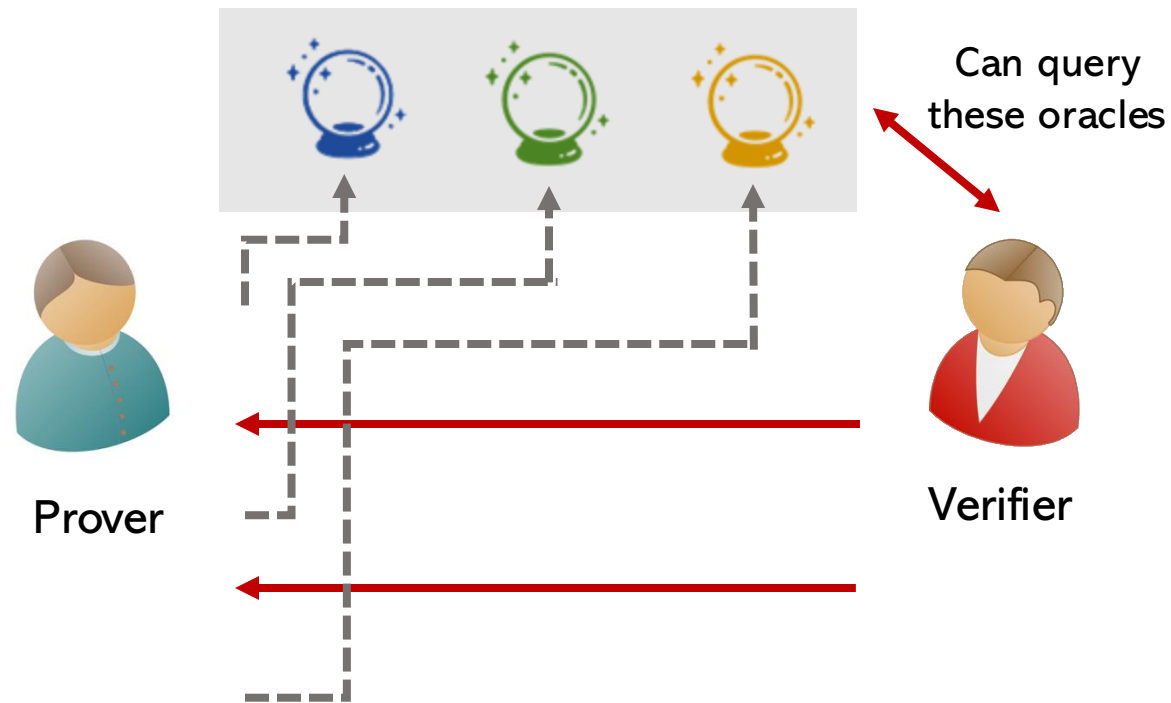


The overheads from FHE and SNARKs will still get multiplied.

$$\begin{aligned} \text{Dec}[\text{Enc}(\pi)] &\rightarrow \pi \\ \text{Dec}[\text{Enc}(\text{out})] &\rightarrow \text{out} \\ \text{Verify}[\pi, F, \text{in}, \text{out}] & \\ \text{Enc}(\text{out}) &= \text{FHE.eval}(F, \text{Enc}(\text{in})) \\ \text{Enc}(\pi) &= \text{FHE.eval}(\text{Prove}, F, \text{Enc}(\text{in}), \text{Enc}(\text{out})) \end{aligned}$$

State-of-the-art SNARKs

Interactive Oracle Proofs → Interactive Proofs → Non-Interactive Proofs



These are information-theoretic primitives

State-of-the-art SNARKs

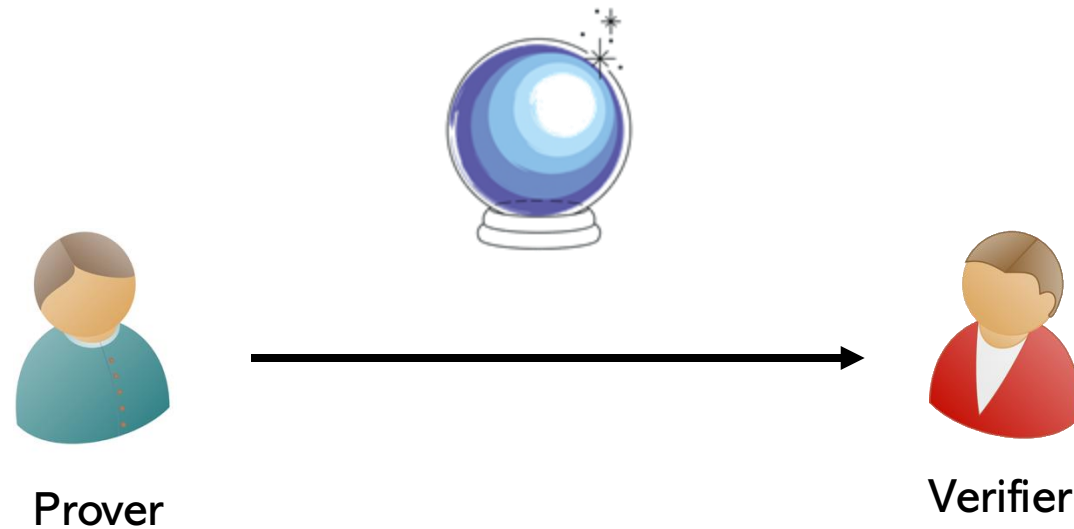
Interactive Oracle Proofs → Interactive Proofs → Non-Interactive Proofs



The transformation to interactive proofs requires the use of cryptography

State-of-the-art SNARKs

Interactive Oracle Proofs → Interactive Proofs → Non-Interactive Proofs



Non-interactive in the random oracle model

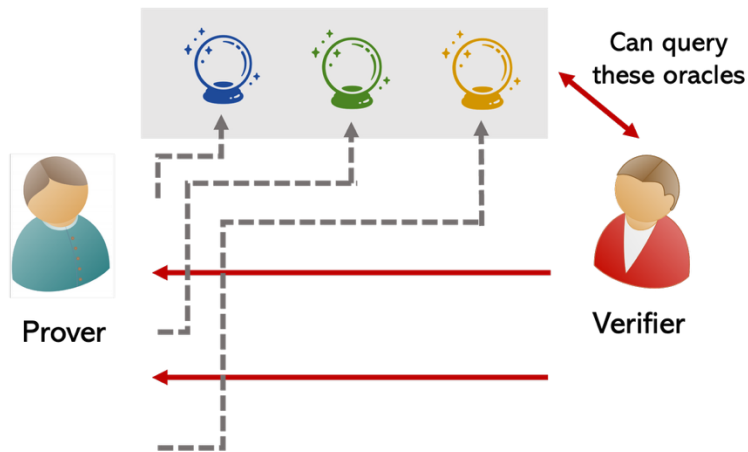
Our Idea

A new compiler for
compiling encrypted IOPs

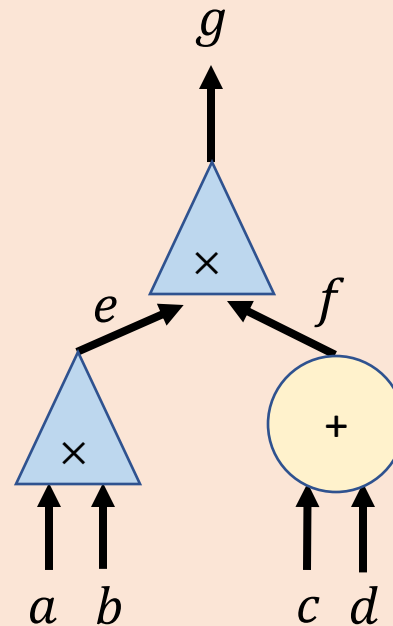
Interactive Oracle Proofs → Interactive Proofs → Non-Interactive Proofs

Encrypted interactive oracle proof

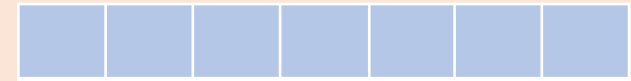
Interactive Oracle Proofs



Represent the function to be proven as a circuit

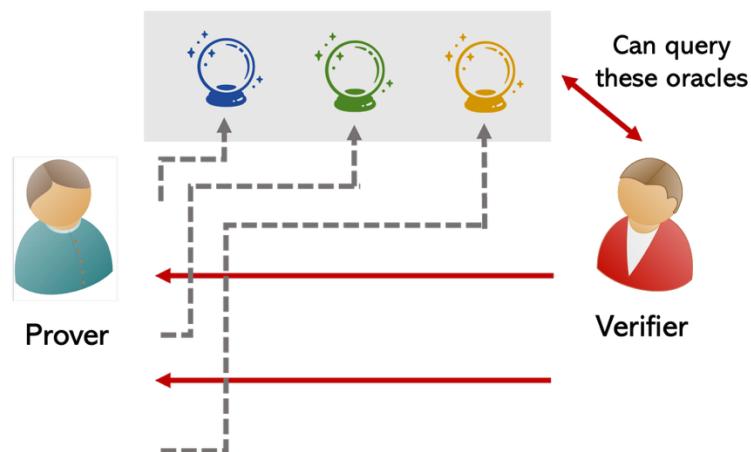


Each oracle can be viewed as a vector of values. The verifier can query linear functions over these values

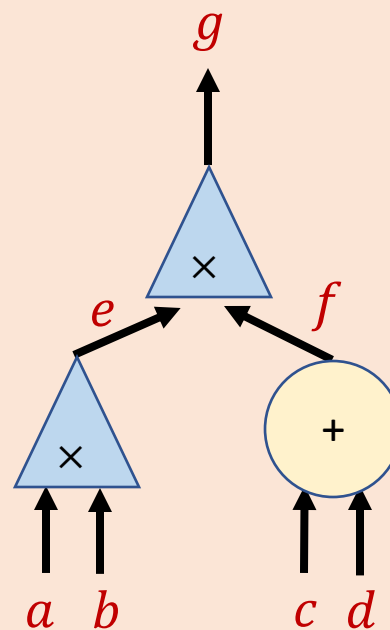


The computation of each oracle is a function of a, b, c, d, e, f, g

Encrypted Interactive Oracle Proofs



Represent the function F as a circuit. The server uses FHE.eval to compute encryptions of all wire values



Use FHE.eval on encrypted values a, b, c, d, e, f, g to compute encrypted oracles

Each oracle is now a vector of encrypted values.



What does the verifier query in the encrypted oracle?

$Enc(\alpha)$	$Enc(\beta)$	$Enc(\gamma)$	$Enc(\delta)$	$Enc(\varepsilon)$	$Enc(\eta)$	$Enc(\vartheta)$
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We observe: The verifier needs to query an encryption of a linear function of the $\alpha, \beta, \gamma, \delta, \dots$ values.

We design: A new cryptographic compiler for this that remains black-box in FHE.

Thanks!



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<https://aarushigoel.github.io/>