

Collaborative zk-SNARKs

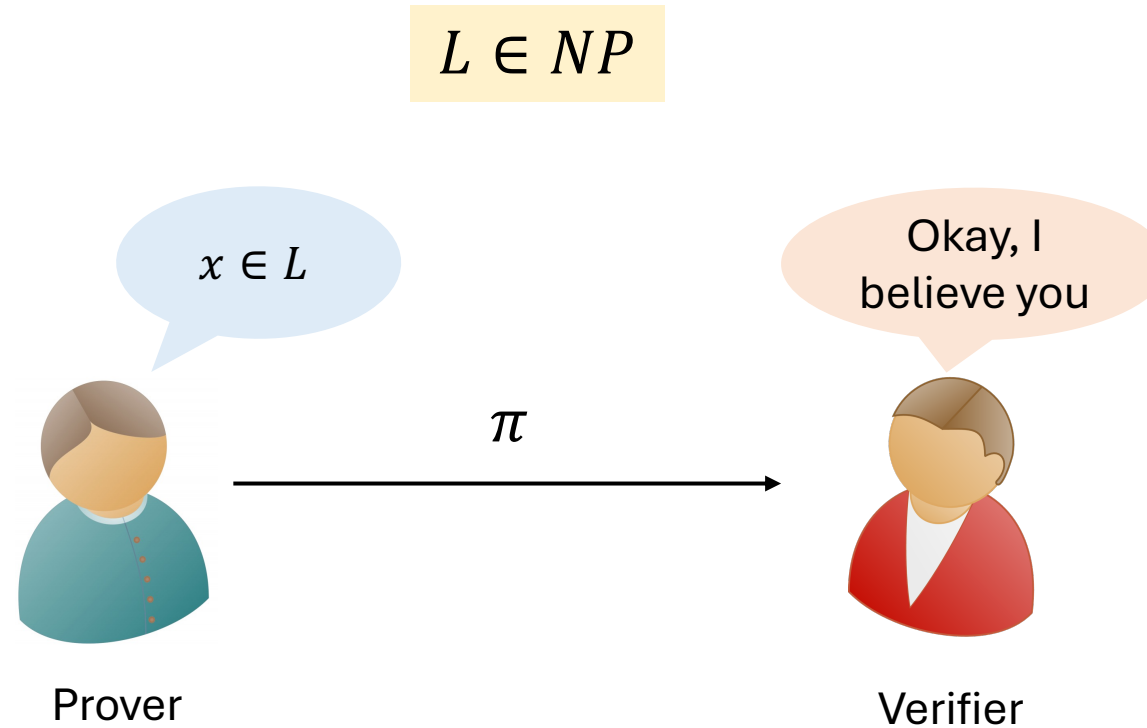
Proving as One over Distributed Secrets

Aarushi Goel



zk-SNARKs

Zero-Knowledge Succinct Non-Interactive Arguments of Knowledge



π is computed by a single prover who knows the witness corresponding to x

What if a single prover does not have the entire witness?

Aggregate Healthcare Statistics

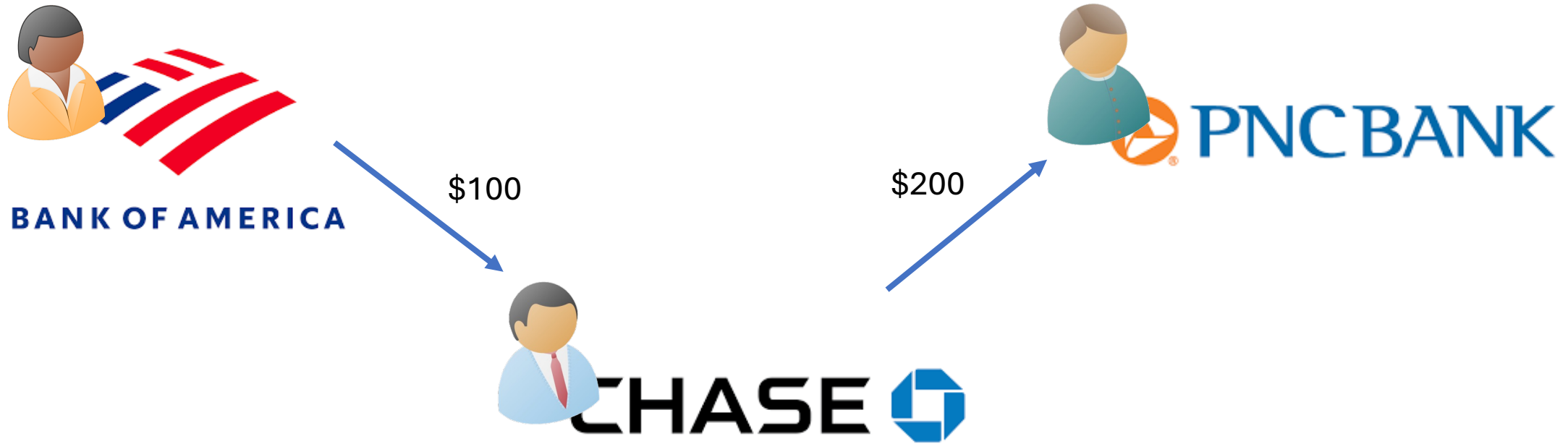


Collectively prove that they do not charge the same price for insured and uninsured patients

Without revealing patient records

What if a single prover does not have the entire witness?

Private Audits of Financial Transactions



Collectively prove whether Alice transferred money to Bob and Bob to Charlie

Without reveal other financial transactions

Common Factors in these Examples

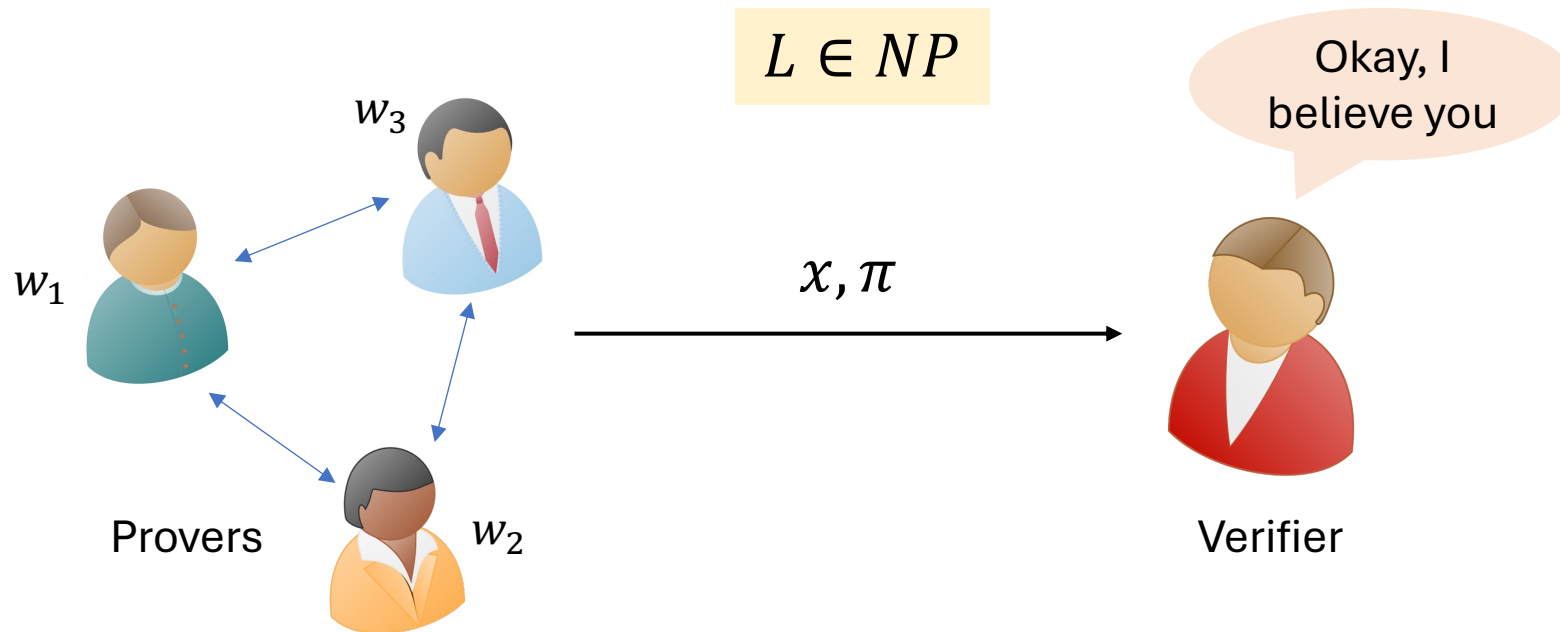
No single entity has the entire witness

No single entity knows whether the statement is true

Multiple entities want to collectively generate a single proof

They do not want to share their private witnesses with each other

Collaborative zkSNARKs [Ozdemir-Boneh'22]



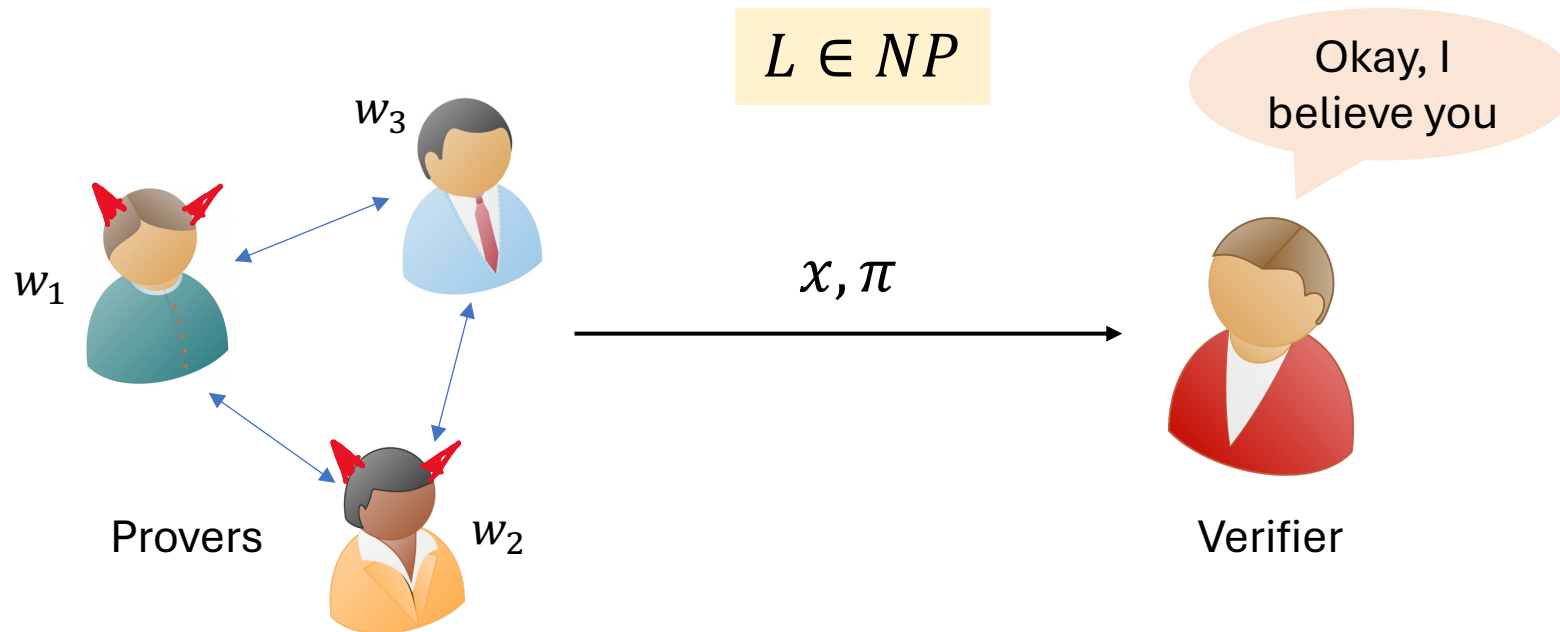
Soundness

π verifies iff $R_L(x, w_1, w_2, w_3) = 1$

Succinctness

π is short

Collaborative zkSNARKs [Ozdemir-Boneh'22]



t-Zero-Knowledge

This is stronger than regular zero-knowledge

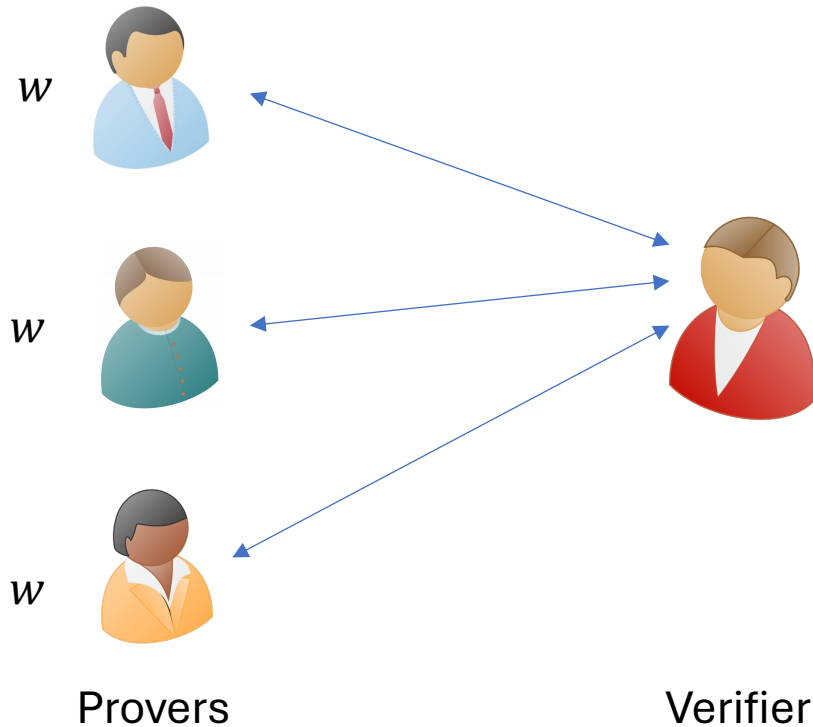
The Verifier and corrupt provers should not learn w_3 even if $x \notin L$



Different from Other Multi-Prover Notions

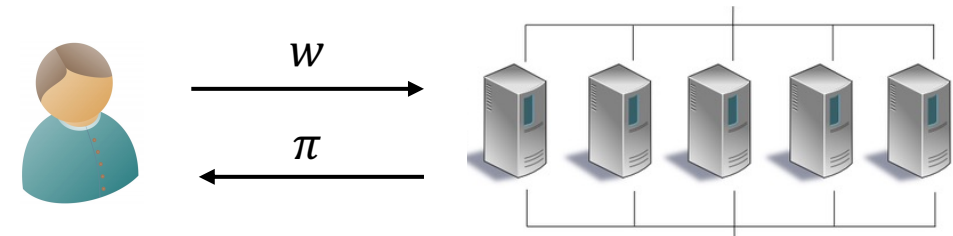
Multi-prover Interactive Proofs (MIPs)

[Ben-Or-Goldwasser-Kilian-Wigderson'88]



Non-privacy preserving distributed proof generation

[Wu-Zhang-Chiesa-AdaPopa-Stoica'18]



Delegating proof computation to a compute cluster

Constructing Collaborative zk-SNARKs



Compute a zk-SNARK using a secure multiparty computation protocol



Where is the non-triviality?

zk-SNARKs are **cryptographic computations**. Computing them using MPC will incur large overheads.

[OB'22] Approach for Constructing Collaborative zkSNARKs

Typical zk-SNARKs



Short witness \rightarrow Extended witness
(Simple **Field Operations**)

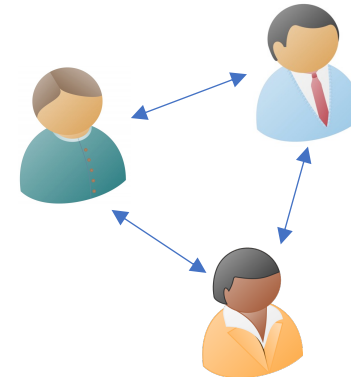


Compute proof using extended witness
(**Cryptographic Operations** + **Field Operations**)

Collaborative zk-SNARKs



Generic MPC for generating extended witness



Custom MPC for computing proof given
extended witness

[OB'22] Approach for Constructing Collaborative zkSNARKs

Typical

This MPC must check validity of joint witness before proceeding with proof computation.

Short witness → Extended witness
(Simple Field Operations)



Else, loss of input privacy!!

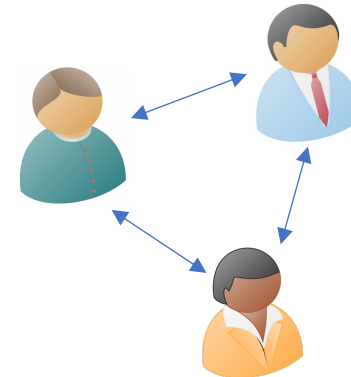
[Garg-G-Jain-Sekar-Roberts 25]

Compute proof using extended witness
(Cryptographic Operations + Field Operations)

Collaborative zk-SNARKs

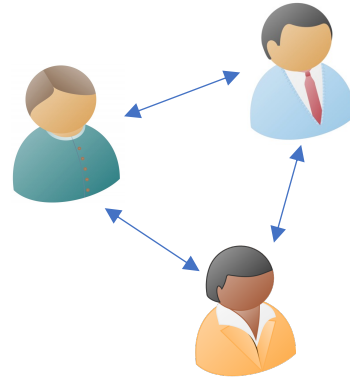


Generic MPC for generating extended witness



Custom MPC for computing proof given extended witness

Rest of this talk will focus on



Custom MPC for computing proof given
extended witness

Custom MPC for computing proof given extended witness

Main novelty of all
prior work lies here



MPC Secure against a
Semi-honest Adversary

Any Existing Malicious
Security Compiler



MPC Secure against a
Malicious Adversary

Custom MPC for computing proof given extended witness

(MPC Secure against a Semi-honest Adversary)



Prior Work: Designing efficient MPC for functions of the following form

Multi-Scalar Multiplications (MSM)

$$F(g_1, \alpha_1, \dots, g_m, \alpha_m) = \prod_{i \in [m]} g_i^{\alpha_i}$$

Fast Fourier Transform (FFT)

For converting between coefficient
and evaluation representation of
polynomials

Partial Products

$$F(x_1, \dots, x_m) = \left(\prod_{i \in [j]} x_i \right)_{j \in [m]}$$

Polynomial Multiplication and Division

A combination of addition,
multiplication and FFT operations

Custom MPC for computing proof given extended witness

(MPC Secure against a Semi-honest Adversary)

Academia



[Ozdemir-Boneh'22]

[Garg-G-Jain-Policharla-Sekar'23]

Multi-Scalar Multiplications (MSM)

[Liu-Zhou-Wang-He-Zhang-Yang-Zhang'24]

$$F(g_1, \alpha_1, \dots, g_m, \alpha_m) = \prod_{i=1}^m g_i^{\alpha_i}$$

[Liu-Zhou-Wang-Zhang-Yang'24]

[Garg-G-Kolonelos-Sinha'25]

For converting between coefficient
and evaluation representation of
polynomials

Industry



Partial Products



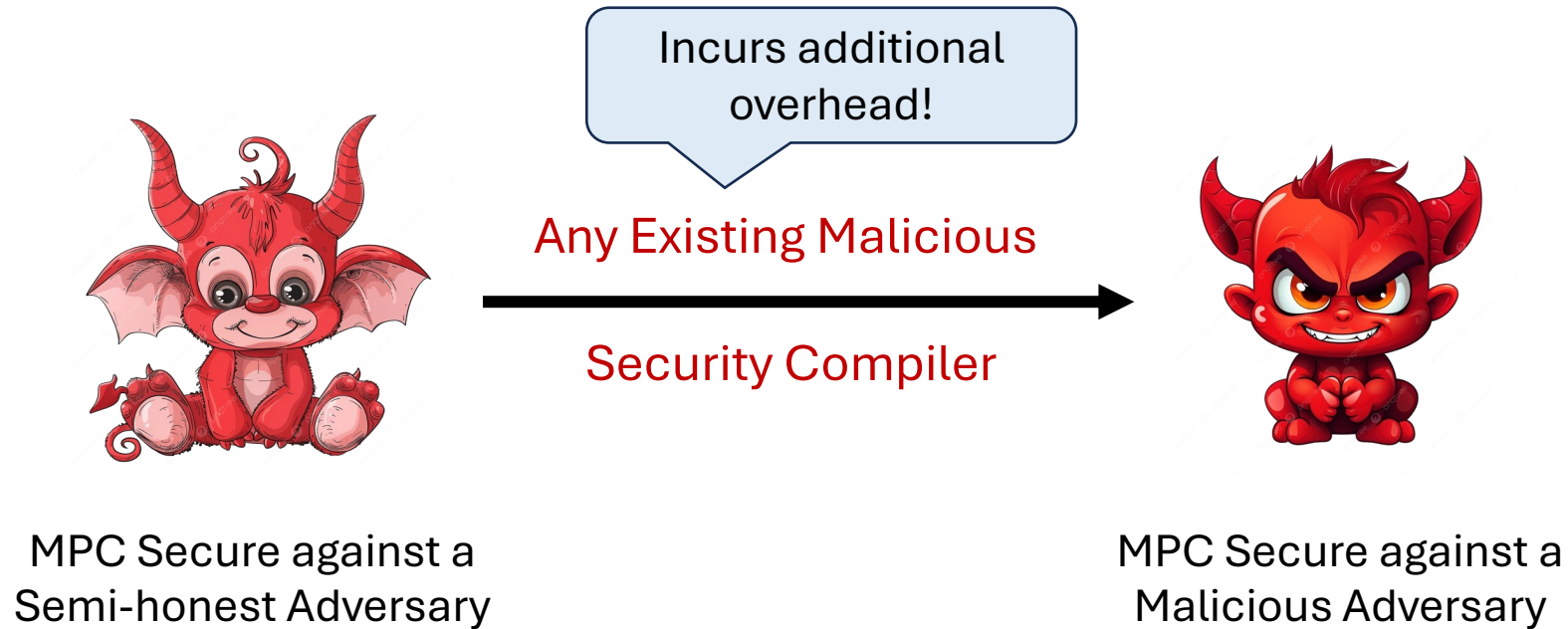
[m]

Polynomial Multiplication and Division



tion,
m operations

Custom MPC for computing proof given extended witness



Can we minimize (or eliminate this overhead?)

[Garg-G-Jain-Sekar-Roberts'25]



Semi-honest
Collaborative zk-SNARK

=



Maliciously Secure
Collaborative zk-SNARK

Additive Attack Paradigm

[Genkin-Ishai-Prabhakaran-Sahai-Tromer'14]

Honest majority
setting

Many secret-sharing based semi-honest MPC protocols are also reasonably secure against malicious adversaries.

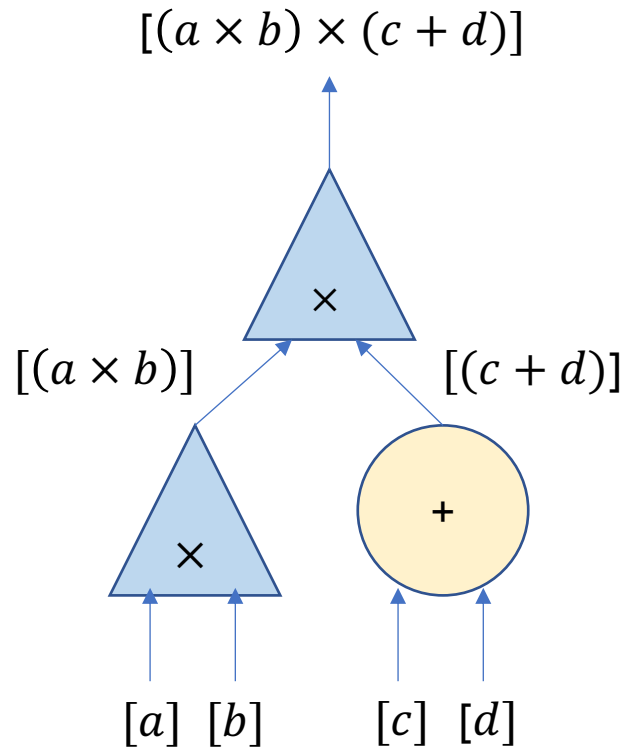


Additive Attack Paradigm

[Genkin-Ishai-Prabhakaran-Sahai-Tromer'14]

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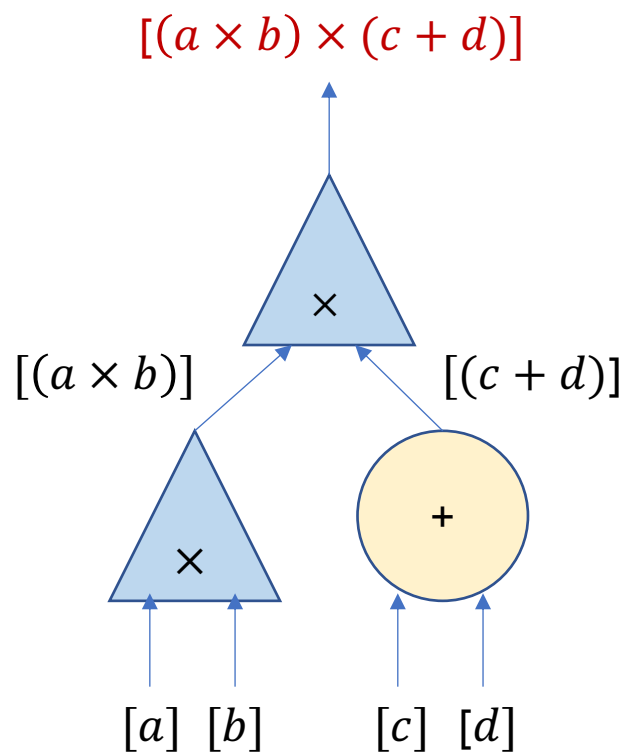


Additive Attack Paradigm

[Genkin-Ishai-Prabhakaran-Sahai-Tromer'14]

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1

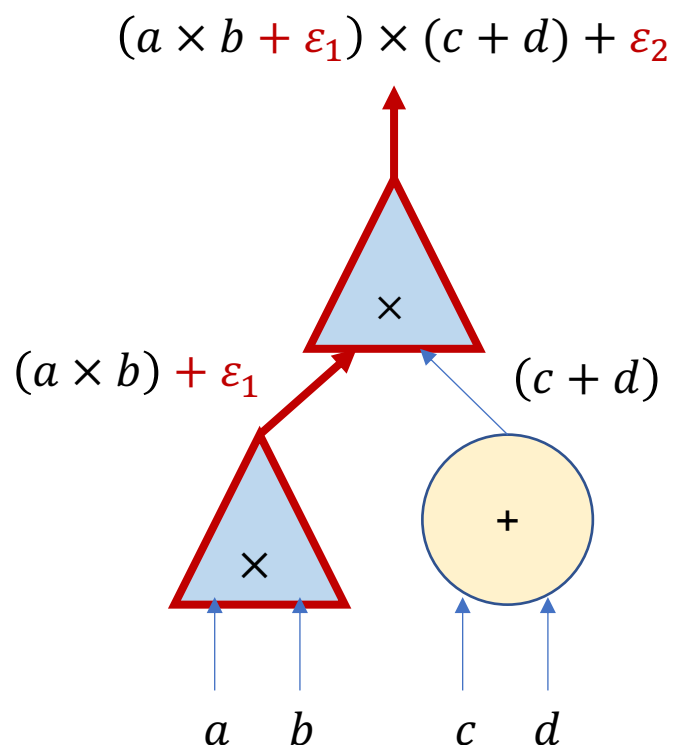
Private until output reconstruction

Additive Attack Paradigm

[Genkin-Ishai-Prabhakaran-Sahai-Tromer'14]

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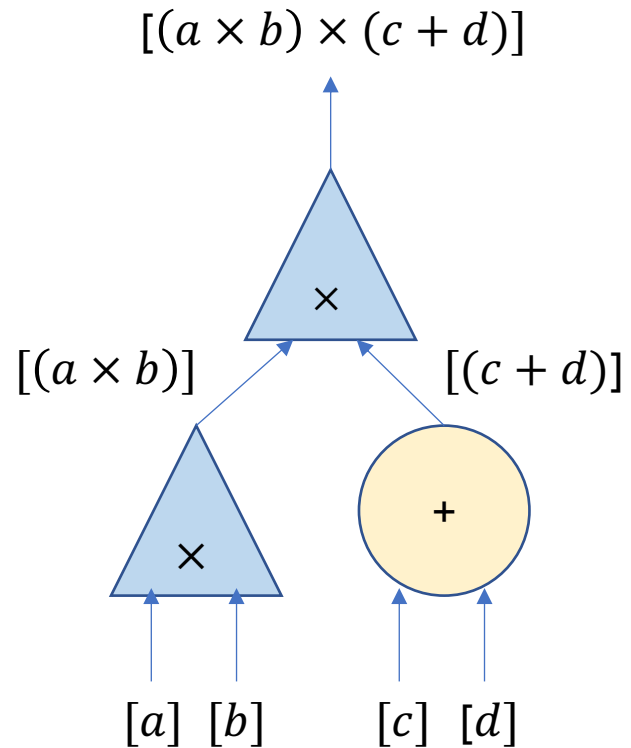
1

Private until output reconstruction

2

Injecting arbitrary additive errors

Semi-Honest \rightarrow Malicious Security (General Strategy)



Use a semi-honest MPC to compute
secret shares of the output

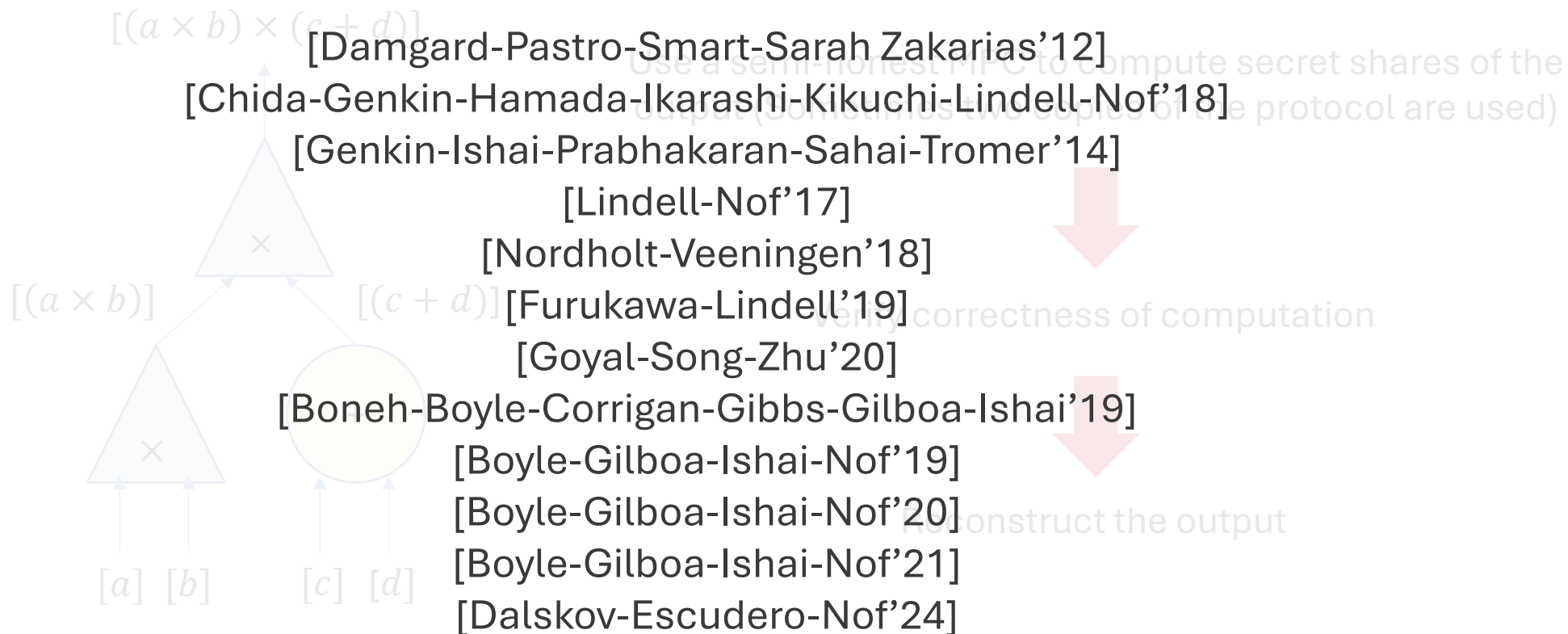


Verify correctness of computation



Reconstruct the output

Semi-Honest → Malicious Security



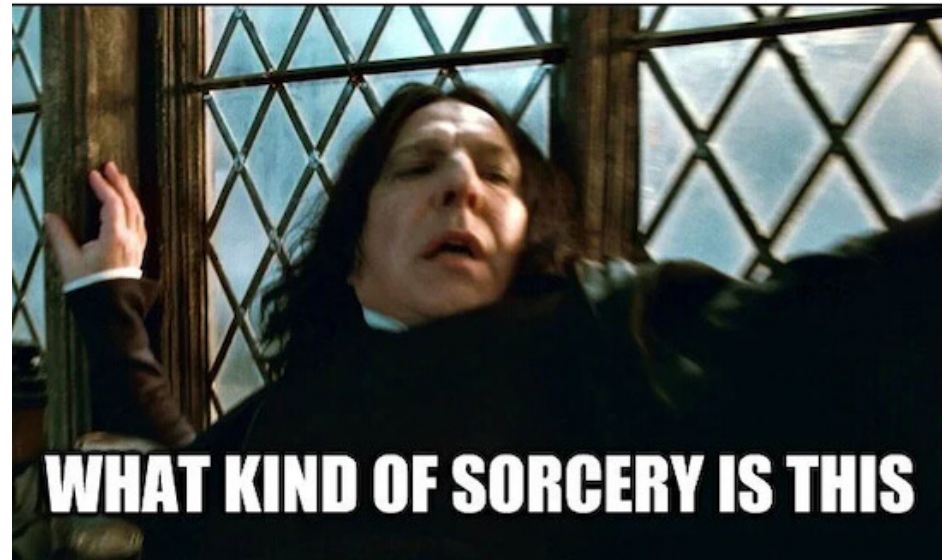


Can we get malicious security for free?

In the honest-majority setting



[Garg-G-Jain-Sekar-Roberts'25]



A man with dark hair, wearing a dark suit and a white shirt, is looking out of a window. The window has a prominent diamond-shaped metal frame. He has a questioning or confused expression on his face, with his hands raised slightly. The background outside the window is a bright, hazy sky.

If the adversary can **inject errors**, how can we even hope to get output correctness?

WHAT KIND OF SORCERY IS THIS



We want to compute zk-SNARKs using MPC

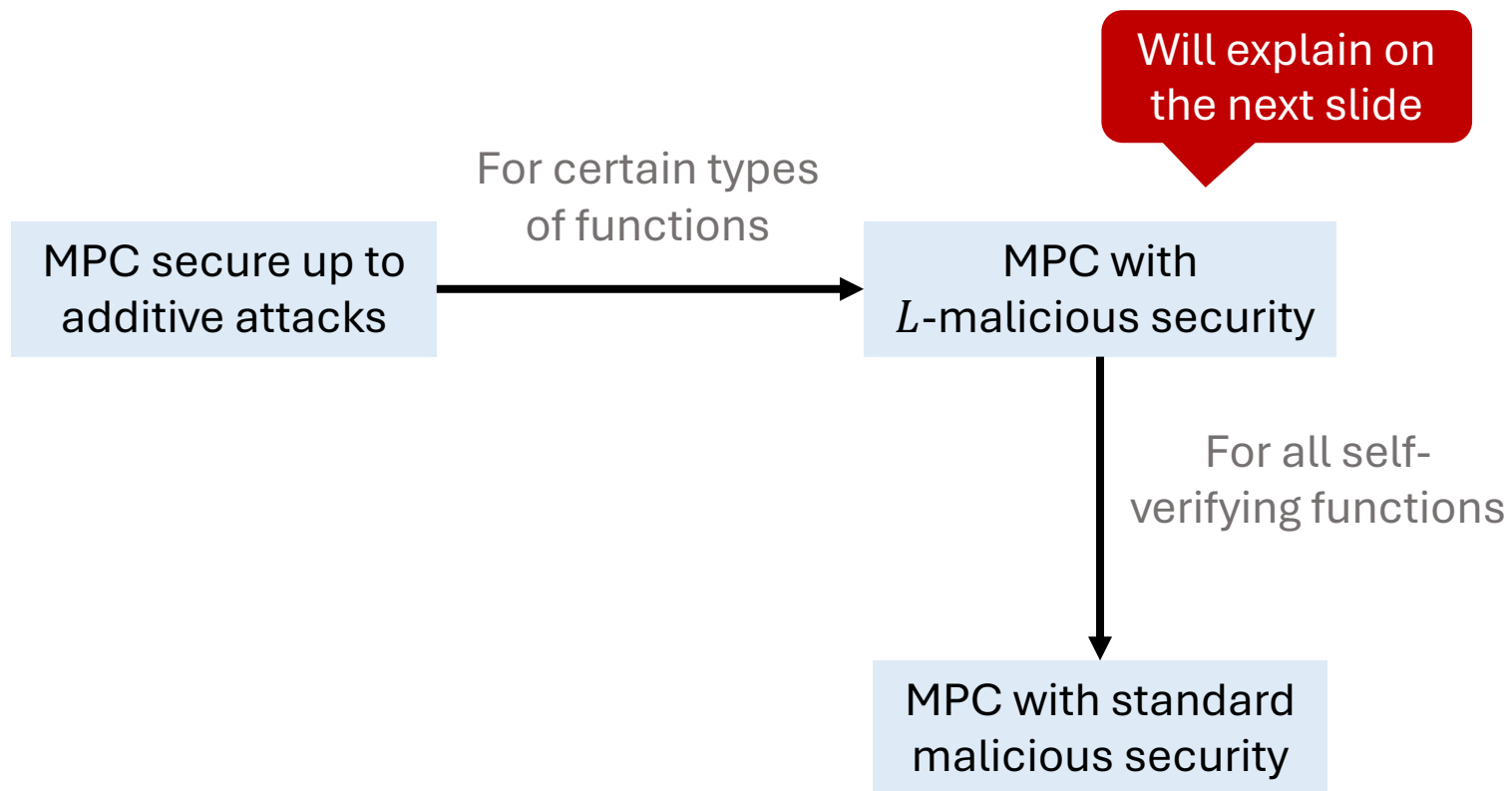


Proofs are self-verifying

MPC does not need to enforce correctness of computed proof
We can simply check whether the proof successfully verifies

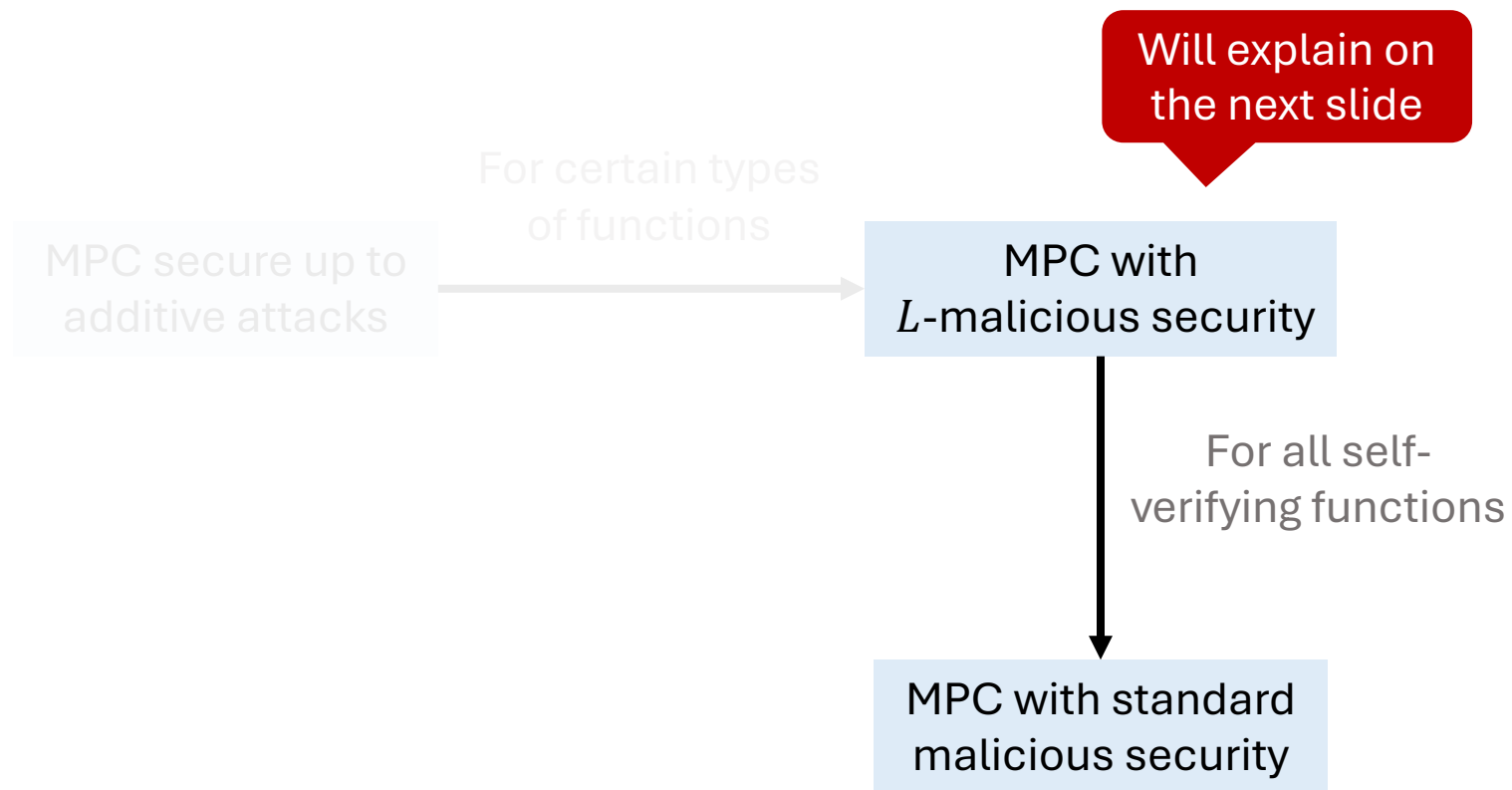


Our Idea



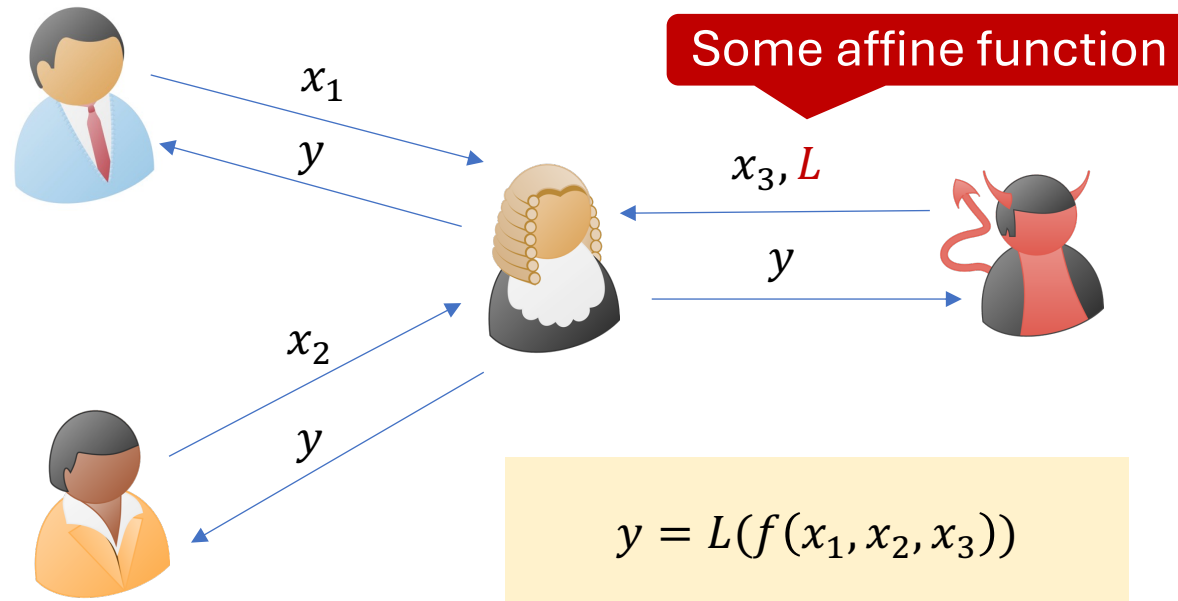


Our Idea



Relaxed Notion of Maliciously Secure MPC

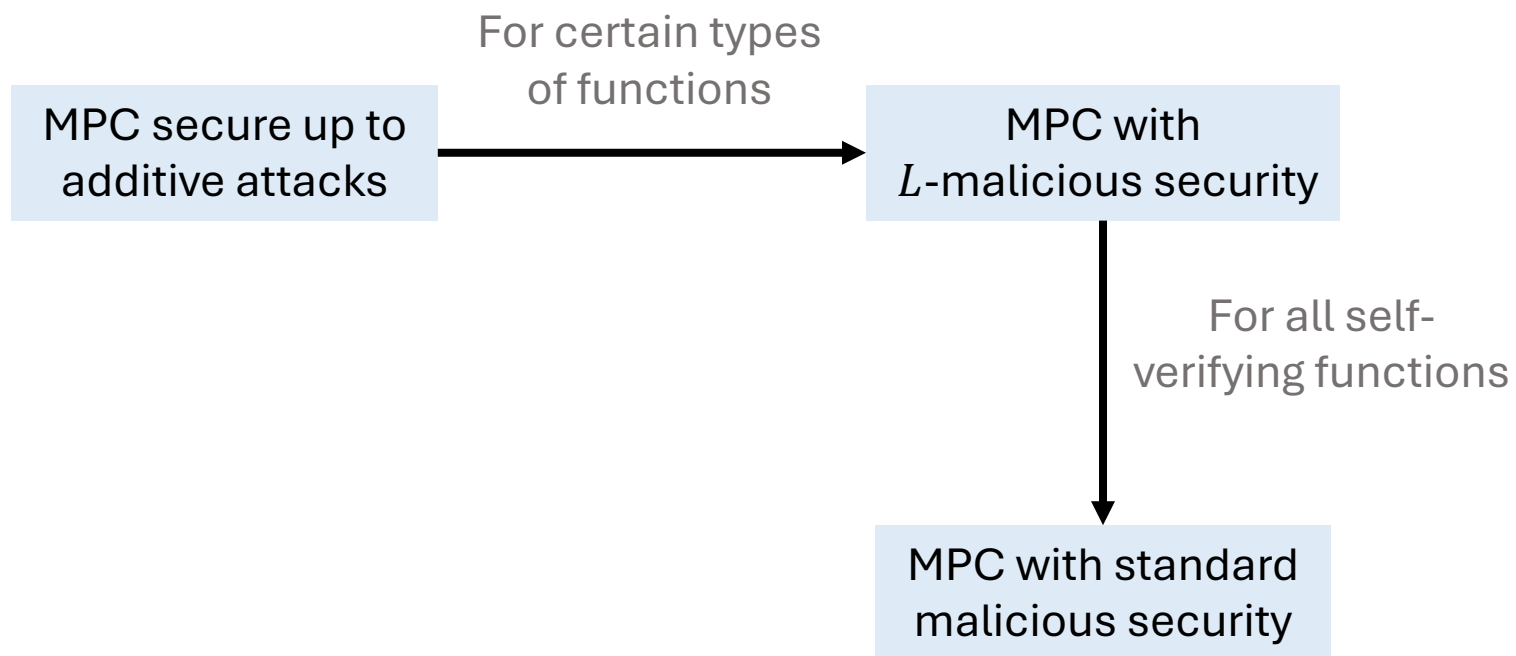
L -Malicious Security



When computing proofs, an L -Maliciously Secure MPC implies standard malicious security

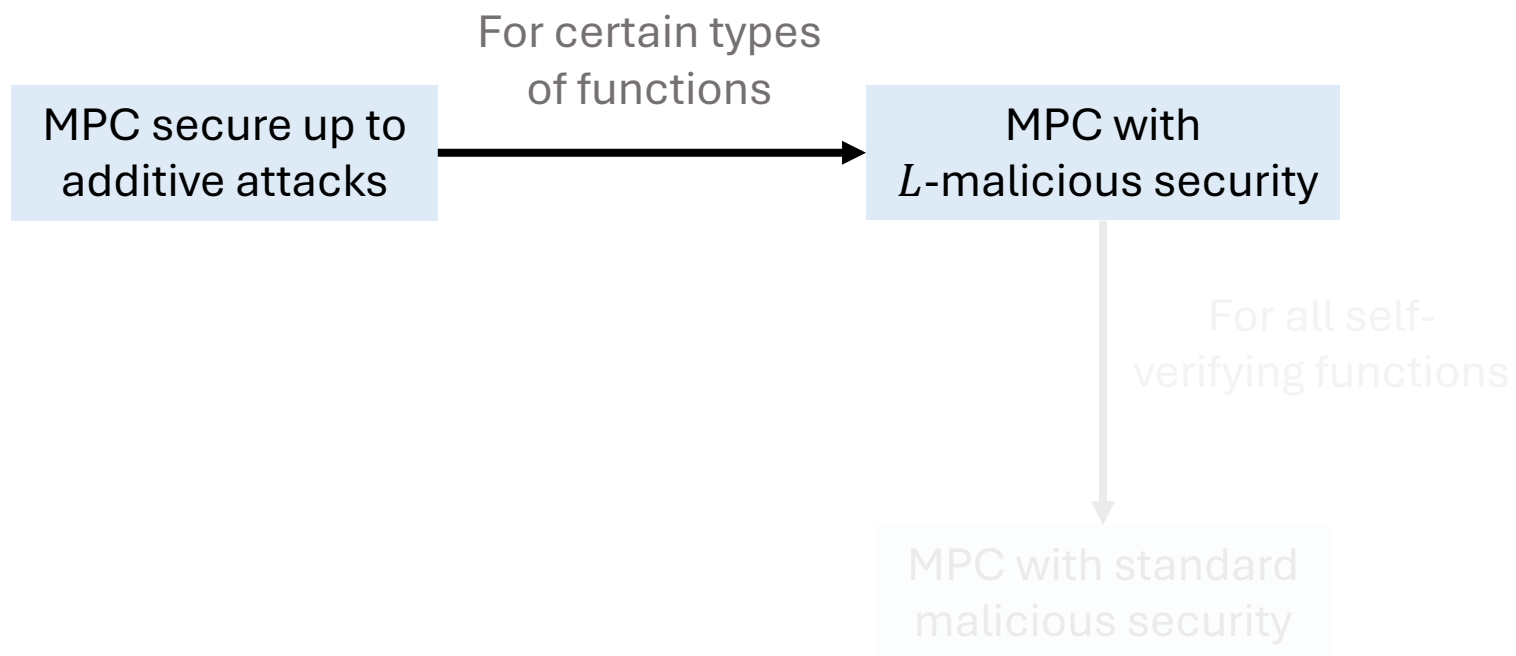


Our Idea





Our Idea



Example 1: Affine Functions

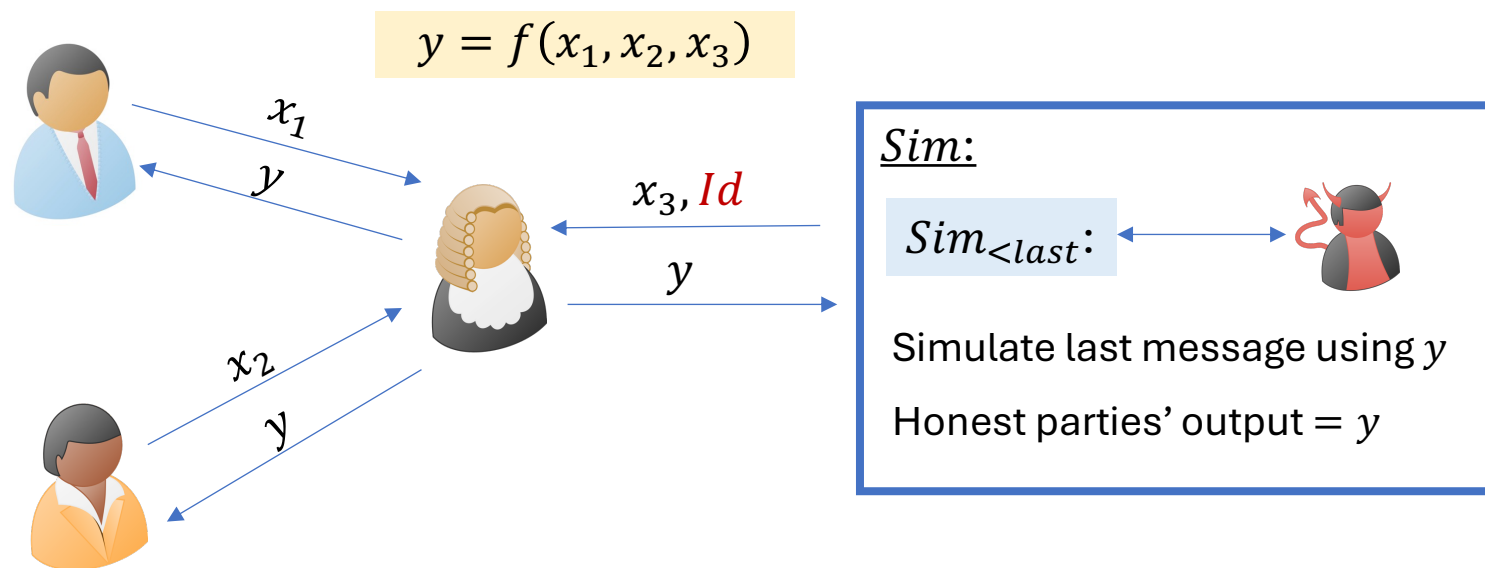
$$f_{A,B,C}(x_1, x_2, x_3) = A \cdot x_1 + B \cdot x_2 + C \cdot x_3 + D \quad \text{No errors!}$$

Assuming x_1, x_2, x_3 are secret shared, while A, B, C, D are constants

How to Simulate?

$Sim_{<last}$:

From [GIPST14], this simulator extracts corrupt inputs and errors while generating an indistinguishable view for all but the last round.



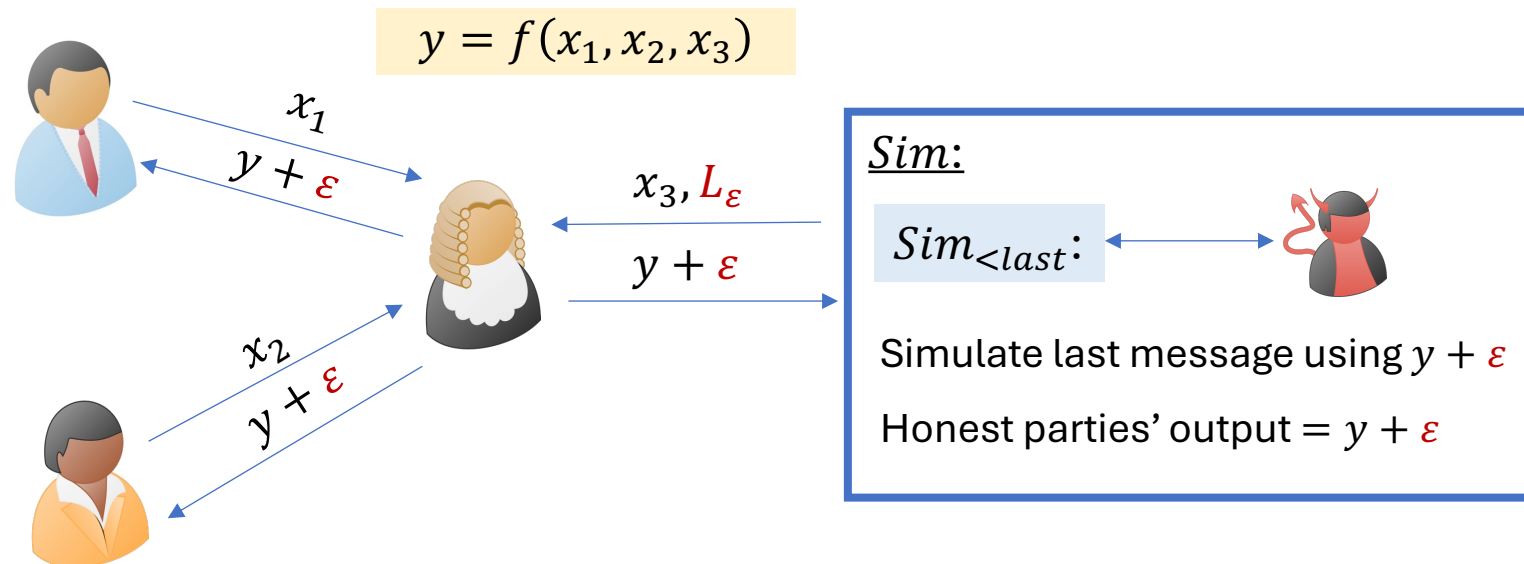
Example 2: Degree-Two Computations

$$f_{A,B}(x_1, x_2, x_3) = x_1 \cdot x_2 + A \cdot x_3 + B$$

Assuming x_1, x_2, x_3 are secret shared, while A, B are constants

Additive error ϵ
can be introduced

How to Simulate?





How is an MPC for such simple functions useful?

Computation of [Groth'16] Proofs can be expressed as a degree-2 computation over the extended witness



t – Zero-Knowledge is different from standard malicious security

In t –zero-knowledge, the simulator does not have oracle access to the proof functionality.
It only learns a bit indicating validity of joint witness.

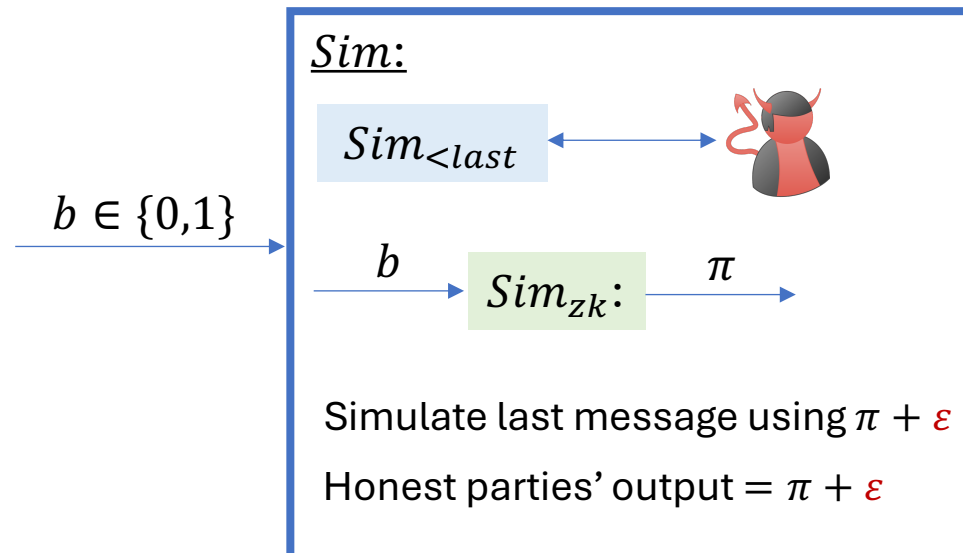
Maliciously Secure Collaborative [Groth16] Proof Generation

Sim_{zk} : this simulator exists from zero-knowledge property of Groth16 zk-SNARKs

How to Simulate?

$$b \stackrel{?}{=} R_L(x, w_1, w_2, w_3)$$

(Determined by the MPC for generating extended witness)



Example 3: Special Randomized Functions

$$f(x_1, x_2; r) = r \cdot (x_1 \cdot x_2)$$

Assuming x_1, x_2 are arbitrary secret shared inputs,
while r is a secret shared random input

Honestly computed output

$$r \cdot (x_1 \cdot x_2)$$

Adversarially computed output

$$r \cdot (x_1 \cdot x_2 + \epsilon_1) + \epsilon_2$$

Both are uniformly distributed.

As a result, given $Sim_{<last}$, the adversary's view in the last round of the MPC can be easily simulated

Example 4: Randomized Encodings

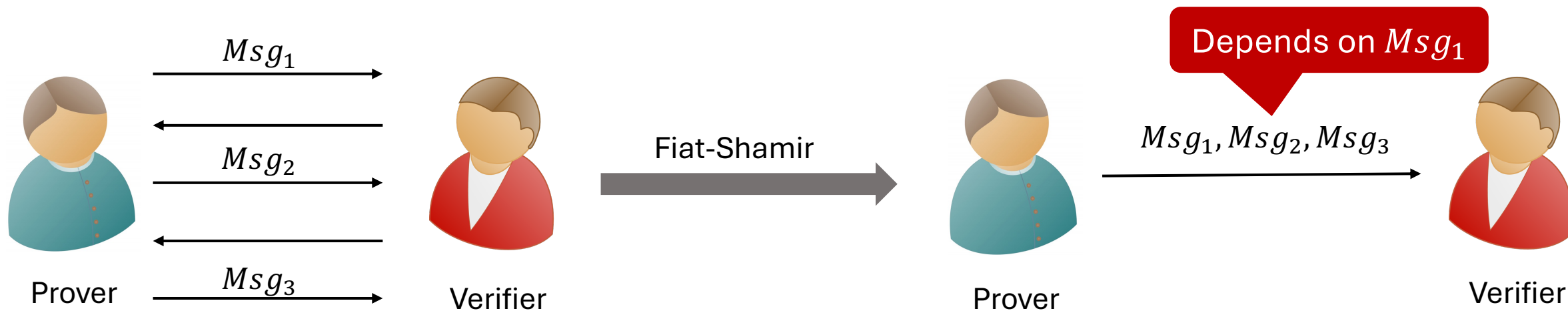
$$f(x_1, x_2, x_3, x_4) = x_1 \cdot x_2 \cdot x_3 \cdot x_4$$

Sequential multiplication over secret shared values

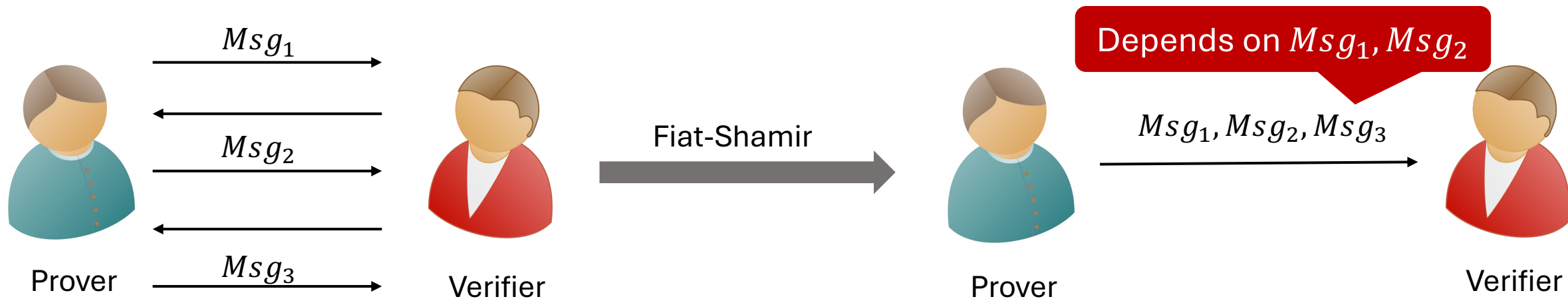
[Bar-Ilan-Beaver'89]: Constant round MPC for sequential multiplications.

Reduces to computing special randomized functions over secret shared values

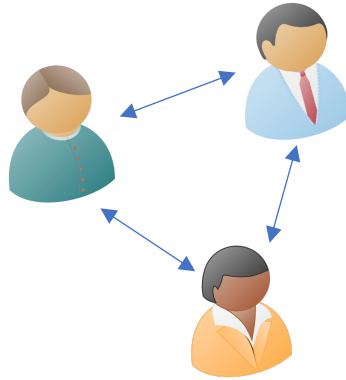
zk-SNARKs in ROM



zk-SNARKs in ROM



Collaborative Variants of zk-SNARKs in ROM



Provers

Run an MPC to compute Msg_1

Reconstruct Msg_1 to query RO

Run an MPC to compute Msg_2

Reconstruct Msg_2 to query RO

-
-
-

If each Msg_i can be computed using one of the example functions, then this overall yields an L – Maliciously secure MPC for reactive functions

Our Results

Bulletproofs: Short Proofs for Confidential Transactions and More

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Andrew Poelstra^{§3}, Pieter Wuille^{¶3}, and Greg Maxwell^{||}

¹Stanford University

²University College London

³Blockstream

Full Version**

Abstract

We propose Bulletproofs, a new non-interactive zero-knowledge proof protocol with very short proofs and without a trusted setup; the proof size is only logarithmic in the witness size. Bulletproofs are especially well suited for efficient range proofs on committed values: they enable proving that a committed value is in a range using only $2 \log_2(n) + 9$ group and field elements, where n is the bit length of the range. Proof generation and verification times are linear in n .

Collaborative Version:



*P*lonK: Permutations over Lagrange-bases for Oecumenical Noninteractive arguments of Knowledge

Ariel Gabizon*
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Aztec

Oana Ciobotaru
Pi Squared

July 17, 2025

Abstract

zk-SNARK constructions that utilize an updatable universal structured reference string remove one of the main obstacles in deploying zk-SNARKs[GKM⁺]. The important work of Maller et al. [MBKM] presented Sonic - the first potentially practical zk-SNARK with fully succinct verification for general arithmetic circuits

Collaborative Version: [Ozdemir-Boneh'22]

We can get free malicious security for these collaborative zk-SNARKs

Open Problems

- What other functions can be computed using semi-honest MPC with free malicious security?
- Can these ideas be extended to the dishonest-majority setting?
- Exploring other types of collaborative zk-SNARKs (e.g., those based on packed secret-sharing).
- Investigating applications of collaborative zk-SNARKs.

Thanks!